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An Improved Hybrid Method for Inverse Obstacle Scattering Problems*

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Abstract: An improved hybrid method is introduced in this paper as a numerical method to reconstruct the scatterer by far-field pattern for just one incident direction with unknown physical properties of the scatterer. The improved hybrid method inherits the idea of the hybrid method by Kress and Serranho which is a combination of Newton and decomposition method, and it improves the hybrid method by introducing a general boundary condition. The numerical experiments show the feasibility of this method.

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1 Introduction

In many applications such as medical imaging and radar, non-destructive obstacle testing through low-frequency wave propagation is necessary. Mathematically, the scattering of time-harmonic acoustical waves by an infinite long cylindrical obstacle with a cross section $D \in \mathbf{R}^2$ leads to two-dimensional exterior boundary value problems. We consider $D \in \mathbf{R}^2$ to be an open bounded obstacle with a C^2-smooth boundary $\Gamma := \partial D$ and an unbounded and connected complement. Then, given an incident field u^i, the direct obstacle scattering problems is to find the total field $u := u^i + u^s$, where u^s is the scattered field, such that u solves the Helmholtz equation in the exterior of the obstacle, i.e.,

$$\Delta u + k^2 u = 0 \qquad \text{in } \mathbf{R}^2 \setminus \bar{D} \tag{1.1}$$

for a wave number $k > 0$, and the boundary condition

$$B(u) = 0 \qquad \text{on } \Gamma. \tag{1.2}$$

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To ensure well-posedness of the scattering problem, one has to impose the Sommerfeld radiation condition

$$\lim_{r\to\infty} r^{1/2}\left(\frac{\partial u^s}{\partial r} - \mathrm{i}ku^s\right) = 0, \qquad r = |x| \tag{1.3}$$

uniformly in all directions. The operator B in (1.2) defines the boundary condition and is related to the physical properties of the obstacle D. The most frequently occurring boundary conditions are the Dirichlet boundary condition

$$B(u) := 0 \qquad \text{on } \Gamma \tag{1.4}$$

for a sound-soft scatter, and the impedance boundary condition

$$B(u) := \partial u/\partial\nu + \mathrm{i}\lambda u \qquad \text{on } \Gamma \tag{1.5}$$

for an impedance obstacle with the exterior unit normal vector ν to Γ and some real-valued impedance function $\lambda \geq 0$ on Γ. The Neumann boundary condition for sound-hard scatterers is included in the impedance boundary condition with $\lambda = 0$.

It is known that the scattered field u^s has an asymptotic behavior of the form

$$u^s(x) = \frac{\mathrm{e}^{\mathrm{i}k|x|}}{\sqrt{|x|}}\left(u_\infty(\hat{x}) + O\left(\frac{1}{|x|}\right)\right), \qquad |x| \to \infty,$$

where

$$\hat{x} = x/|x|.$$

The function u_∞ is known as the far field pattern of u^s and is analytical on the unit circle Ω.

The inverse scattering problem that we are concerned with is to determine the shape and location of the scatterer D from a knowledge of the far field pattern u_∞ for one incident plane wave. The inverse problem is non-linear and ill-posed, which makes the solution difficult. For the uniqueness, see [1]. Although there is widespread belief that the far field pattern for one single direction and one single wave number determines the scatterer without any additional a priori information, establishing this result still remains a challenging open problem.

Many methods have been proposed to numerically solve the inverse scattering problem with known boundary condition, such as the decomposition method by Kirsch and Kress (see [2]–[4]), the Newton method (see [5] and [6]), the hybrid method by Kress and Serranho (see [7]–[9]) and some other iterative methods based on integral equations (see [10]–[12]). We are interested in the hybrid method which combines the advantages of both decomposition methods and Newton iterations. The hybrid method has been successfully used in the case of sound-soft, sound-hard and impedance obstacles. We gives a general overview over this method by two steps. For details of this method, we refer to [7]–[9].

We denote by $\Gamma^n := \partial D_n$ the approximation to the boundary Γ at the n-th iteration given by this method. In the first step, the scattered field u^s is represented in the form of a combined acoustic double- and single-layer potential over the closed C^2-contour Γ^n :

$$u^s = \int_{\Gamma^n}\left\{\frac{\partial\Phi(x,y)}{\partial\nu(y)} - \mathrm{i}\eta\Phi(x,y)\right\}\varphi(y)\mathrm{d}s(y), \qquad x \in \mathbf{R}^2 \setminus \bar{D}_n \tag{1.6}$$