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Contact Finite Determinacy of Relative Map Germs*

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Abstract: The strong contact finite determinacy of relative map germs is studied by means of classical singularity theory. We first give the definition of a strong relative contact equivalence (or $\mathcal{K}_{S,T}$ equivalence) and then prove two theorems which can be used to distinguish the contact finite determinacy of relative map germs, that is, f is finite determined relative to $\mathcal{K}_{S,T}$ if and only if there exists a positive integer k, such that $\mathcal{M}^k(n)\mathcal{E}(S;n)^p \subset T\mathcal{K}_{S,T}(f)$.

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1 Introduction

A basic idea in the classical singularity theory is that local topological properties of a generic differentiable mapping are determined by finite terms of its Taylor series, i.e., finite determinacy of map germs. It relates to the most important local characteristics of the singularity theory. Therefore, finite determinacy is always an active research subject in the singularity theory. When we treat various spaces of differentiable mappings with several constraints, depending on situations, we need to study the corresponding finite determinacy, whose validity depends on given mapping spaces.

In the present paper we treat the space of differentiable mappings between manifolds with the constraint that a fixed submanifold is mapped into another fixed submanifold. Then naturally we need to study the "relative finite determinacy".

There are several works studying relative finite determinacy of function germs, for instance [1]–[7]. However, the study of relative finite determinacy of map germs such as [8]

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is very few. This paper is a sequel to [8]. The purpose of the present paper is to give an algebraic criterion for relative contact finite determinacy of relative smooth map germs. It is a generalization of the algebraic criteria for $\mathcal{K}$ finite determinacy of smooth map germs originated in [9].

This paper is organized as follows: In Section 2 we give some notations, definitions and other related knowledge. Section 3 is the main part of this paper. We prove the main results in a similar way to [9].

2 Preliminaries

Let S, T be submanifolds without boundary of $\mathbf{R}^n$ and $\mathbf{R}^p$ respectively, both contaning the origin. Denote by $M(R^n, S; R^p, T)$ the set of relative smooth mappings $f : (\mathbf{R}^n, S) \to (\mathbf{R}^p, T)$ with $f(S) \subset T$ and $f(0) = 0$. For $f, g \in M(\mathbf{R}^n, S; \mathbf{R}^p, T)$, we call f, g equivalent near the origin if $f = g$ on some neighborhood of the origin in R^n. This equivalence class of f is called a relative map germ and denoted by $[f]$. In this paper we also denote it by f for convenience. We only consider the local case, so we may assume that

$$S = \{0\} \times \mathbf{R}^{n-s}, \qquad T = \{0\} \times \mathbf{R}^{p-t}.$$

Denote by $M^* = M(n, s; p, t)$ the set of relative map germs and ${E_f}^*{=}E(f, n, s; p, t)$ the set of relative map germs g which satisfies $g(S) = f(S)$ for a given $f \in M^*$. Let

$$C_S(\mathbf{R}^n) = \{h : (\mathbf{R}^n, S) \to R \mid h|_S = \text{Constant}\}$$

be a local ring and

$$\mathcal{E}(S; n) = \{h \in C_S(\mathbf{R}^n) \mid h|_S = 0\}$$

be the maximal ideal of $C_S(\mathbf{R}^n)$. Similarly, we can define the set $C_T(\mathbf{R}^p)$ and $\mathcal{E}(T; p)$. For $f \in M^*$, it induces a homomorphism $f^* : C_T(\mathbf{R}^p) \to C_S(\mathbf{R}^n)$ defined by $f^*(h) = h \circ f$ with $f^*\mathcal{E}(T; p) \subseteq \mathcal{E}(S; n)$. Every $C_S(\mathbf{R}^n)$-module can be viewed as a $C_T(\mathbf{R}^p)$-module through f^*. We denote

$$L_n = \{h : (\mathbf{R}^n, 0) \to (\mathbf{R}^n, 0) \mid h \text{ is a diffeomorphism germ at the origin}\}$$

and

$$\mathcal{K} = \{H \mid H \in L_{n+p}\}.$$

We can also denote by

$$\mathcal{K}_{S,T} = \{H \in \mathcal{K} \mid H(x, y) = (x, y),\ \forall x \in S,\ y \in T\}$$

the strong relative contact equivalent group, where $\mathcal{K}_{S,T}$ acts on M^* in a natural way: If $f \in M^*$, $H \in \mathcal{K}_{S,T}$, then $H \cdot f$ is defined by

$$(\mathbf{1}, H \cdot f) \circ h = H \circ (\mathbf{1}, f),$$

where $h : (\mathbf{R}^n, 0) \to (\mathbf{R}^n, 0)$, $h(x) = x$, for any $x \in S$ is a diffeomorphism of $\mathbf{R}^n$ decided only by H and $\mathbf{1}$: $(\mathbf{R}^n, 0) \to (\mathbf{R}^n, 0)$ is the identity map germ. We also observe that $\mathcal{K}_{S,T}$ is a subgroup of the general contact equivalent group $\mathcal{K}$ in [9]. More details can be found in [8] and [9].