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Analysis of a Prey-predator Model with Disease in Prey*

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Abstract: In this paper, a system of reaction-diffusion equations arising in eco-epidemiological systems is investigated. The equations model a situation in which a predator species and a prey species inhabit the same bounded region and the predator only eats the prey with transmissible diseases. Local stability of the constant positive solution is considered. A number of existence and non-existence results about the non-constant steady states of a reaction diffusion system are given. It is proved that if the diffusion coefficient of the prey with disease is treated as a bifurcation parameter, non-constant positive steady-state solutions may bifurcate from the constant steady-state solution under some conditions.

Key words: eco-epidemiology, bifurcation, non-constant positive steady solution

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1 Introduction

Mathematical ecology and mathematical epidemiology are major fields of study. Since transmissible disease in ecological situation cannot be ignored, it is very important from both the ecological and the mathematical points of view to study ecological systems subject to epidemiological factors. A number of studies have been performed in this direction; see [1]–[9] and the references therein. Combining a typical SI model with an open system of variable size and a general predator-prey model, Bairagi et al. proposed a eco-epidemiological model in [10] as follows:

$$\begin{cases} u_t = ru\Big(1 - \dfrac{u+v}{k}\Big) - m_1 uv, \\ v_t = m_1 uv - \dfrac{m_2 vw}{a+v} - m_3 v, \\ w_t = \dfrac{m_4 vw}{a+v} - m_5 w, \\ u(0) > 0, \quad v(0) > 0, \quad w(0) > 0, \end{cases} \tag{1.1}$$

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where u, v and w are the densities of susceptible prey, infected prey and predator, respectively; r, k, m_i, $i = 1, 2, \cdots, 5$ are positive constants; m_1 is the rate of transmission; m_2 is the search rate; m_3 is the death rate of infected prey; m_4 represents the conversion factor; m_5 is the total death of predator population and a is the half saturation coefficient.

This model implies that the prey is divided into two disjoint classes, susceptible prey u and infected prey v. Only susceptible prey has capability of reproducing, but the infected prey still contributes with u to population growth towards the carrying capacity k. Disease transmission follows the simple law of mass action. The disease is spread among the prey population only. The infected population do not recover or become immune. It is assumed that predator consume only infected preys at the rates $m_2v/(a+v)$. For more detailed biological meaning the reader may consult [10].

As we know, most of the eco-epidemiological models are ODE systems. If we take into account the distribution of the species in spatial locations within a fixed bounded domain $\Omega \in \mathbf{R}^N$ with smooth boundary $\partial\Omega$ and both species diffuse, i.e., move from points of high to points of low population density, then (1.1) may be rewritten as

$$\begin{cases} u_t = d_1\Delta u + ru\Big(1 - \dfrac{u+v}{k}\Big) - m_1uv, & x \in \Omega, \quad t > 0, \\ v_t = d_2\Delta v + m_1uv - \dfrac{m_2vw}{a+v} - m_3v, & x \in \Omega, \quad t > 0, \\ w_t = d_3\Delta w + \dfrac{m_4vw}{a+v} - m_5w, & x \in \Omega, \quad t > 0, \\ \partial_n u = \partial_n v = \partial_n w = 0, & x \in \partial\Omega, \ t > 0, \\ u(x,0) \geq 0, \quad v(x,0) \geq 0, \quad w(x,0) \geq 0, & x \in \Omega, \end{cases} \tag{1.2}$$

where ∂_n is the outward directional derivative normal to $\partial\Omega$ and the positive constants d_1, d_2 and d_3 are the diffusion rates. The initial data $u(x,0)$, $v(x,0)$ and $w(x,0)$ are continuous functions on $\bar{\Omega}$. The homogeneous Neumann boundary condition means that (1.2) is self-contained and has no population flux across the boundary $\partial\Omega$.

The positive steady state solutions of (1.2) satisfy the following elliptic system:

$$\begin{cases} d_1\Delta u + ru\Big(1 - \dfrac{u+v}{k}\Big) - m_1uv = 0, & x \in \Omega, \\ d_2\Delta v + m_1uv - \dfrac{m_2vw}{a+v} - m_3v = 0, & x \in \Omega, \\ d_3\Delta w + \dfrac{m_4vw}{a+v} - m_5w = 0, & x \in \Omega, \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial v}{\partial n} = \dfrac{\partial w}{\partial n} = 0, & x \in \partial\Omega. \end{cases} \tag{1.3}$$

For the simplicity of notation, we denote

$$\Lambda = (r, k, m_1, m_2, m_3, m_4, m_5), \qquad U = (u, v, w).$$

We note that (1.2) has a unique nonnegative global solution U which can be proved by using the method of upper and lower solutions. In addition, if $u(x,0) \not\equiv 0$, $v(x,0) \not\equiv 0$, $w(x,0) \not\equiv 0$, then the solution is positive, i.e., $u > 0$, $v > 0$, $w > 0$ on $\bar{\Omega}$ for all $t > 0$. The equation (1.2)