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Dividend Maximization when Cash Reserves Follow a Jump-diffusion Process*

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Abstract: This paper deals with the dividend optimization problem for an insurance company, whose surplus follows a jump-diffusion process. The objective of the company is to maximize the expected total discounted dividends paid out until the time of ruin. Under concavity assumption on the optimal value function, the paper states some general properties and, in particular, smoothness results on the optimal value function, whose analysis mainly relies on viscosity solutions of the associated Hamilton-Jacobi-Bellman (HJB) equations. Based on these properties, the explicit expression of the optimal value function is obtained. And some numerical calculations are presented as the application of the results.

Key words: jump-diffusion model, dividend payment, Hamilton-Jacobi-Bellman equation, viscosity solution

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1 Introduction

The dividend optimization problem is a classical problem in actuarial mathematics. It was first proposed by De Finetti[1] and has been studied by many authors. For the diffusion risk model, the problem was solved by Asmussen and Taksar[2], identifying barrier strategies as the optimal ones and giving the expression of the value of discounted dividends. Other related problems, such as the interplay among risks, dividend and investment policies and the moments and distribution of the discounted dividends paid until ruin, can be seen, e.g., [3]–[7] and references therein. For the classical compound Poisson risk model, the work has not got results as much as that of diffusion models due to the complexity of model structure. Gerber[8] considered the case of discrete model, and Azcue and Muler[9] studied the problem of continuous model, showing band strategies as the optimal ones and describing the value

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of discounted dividends for small initial reserves.

The common approach for studying the optimal value function is to derive the Hamilton-Jacobi-Bellman equation, which is usually an integro-differential or differential equation, under the assumption that the objective function is smooth enough, and then construct its solutions. Explicit solutions may often be derived only for special cases, typically for exponential claim size distributions. However, it is necessary to discuss the smoothness of objective function from a theoretical viewpoint. The similar discussion for the ruin probability can be found in [10]–[12]. And from a practical viewpoint, it is also useful if we can obtain the expression of optimal value function for general cases.

In this paper, we discuss the problem under the assumption that the optimal value function is concave. We start with analyzing the general properties on the optimal value function via the technique of stochastic control and the theory of viscosity solutions. Then we prove the optimal value function $V(x)$ in risk model (2.1) is twice continuously differentiable provided that the claim size has a continuous distribution function. Moreover, we present the closed form solution of $V(x)$ for arbitrary continuous claim size distributions by constructing an identical equation.

The rest of the paper is organized as follows. In Section 2, we give a rigorous mathematical formulation of the problem. In Section 3, we show that the optimal value function is a viscosity solution of the associated HJB equation. Section 4 demonstrates the C^2 smoothness of the optimal value function. Section 5 describes the construction of the optimal value function for arbitrary continuous claim size distributions. In Section 6, we present two numerical examples as the applications of the results.

2 Formulation of the Problem

For a rigorous mathematical formulation, we start with a probability space $(\Omega,\ \mathscr{F},\ P)$, a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ and a process $\{W_t\}_{t\geq 0}$, which is a standard Brownian motion with respect to $\{\mathscr{F}_t\}_{t\geq 0}$. The filtration $\mathscr{F}_t$ represents the information available at time t and any decision is made based upon this information. We assume that the surplus of an insurance company follows the perturbed compound Poisson risk process of Dufresne and Gerber[13], namely,

$$U_t = x + \mu t + \sigma W_t - \sum_{i=1}^{N(t)} Y_i, \qquad t \geq 0.$$

Here $x \geq 0$ is the initial surplus, $\mu > 0$ is the rate of premium, $\sigma > 0$ is the dispersion parameter, $\{N(t),\ t \geq 0\}$ is a Poisson process with parameter $\lambda > 0$, representing the total number of claims up to time t, and the claim size $\{Y_i,\ i = 1, 2, \cdots\}$, independent of $\{N(t),\ t \geq 0\}$ and $\{W(t),\ t \geq 0\}$, is a sequence of independent and identically distributed positive random variables with continuous distribution function P and density function p.

We now enrich the model. We allow the surplus to accrue interest at a constant rate $r \geq 0$, and assume that dividends are paid to the shareholders according to some strategy.