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New Energy Analysis of Yee Scheme for Metamaterial Maxwell's Equations on Non-Uniform Rectangular Meshes

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Abstract. In this paper, several new energy identities of metamaterial Maxwell's equations with the perfectly electric conducting (PEC) boundary condition are proposed and proved. These new energy identities are different from the Poynting theorem. By using these new energy identities, it is proved that the Yee scheme on non-uniform rectangular meshes is stable in the discrete L^2 and H^1 norms when the Courant-Friedrichs-Lewy (CFL) condition is satisfied. Numerical experiments in two-dimension (2D) and 3D are carried out and confirm our analysis, and the superconvergence in the discrete H^1 norm is found.

AMS subject classifications: 65M06, 65M12, 35L15, 78A48

Key words: Metamaterial Maxwell's equations, Yee scheme, non-uniform rectangular meshes, energy identities, stability.

1 Introduction

Metamaterials are artificial composite materials designed to exhibit exotic electromagnetic properties. The metamaterial with negative refraction index was first proposed by Veselago in 1968 [26] and constructed by Smith in 2000 [23,24], which has brought a new revolution in electromagnetic and material science. Since 2000, there are numerous reference sources on the study of metamaterials and their potential applications, such as, design of invisiblity cloak [8,22], sub-wavelength imaging [1,28], construction of perfect lens [25]. Matical side there has recently been increased interest in the understanding of the mathematical properties of metamaterial Maxwell's equations relevant to numerical analysis. For example, finite-difference time-domain (FDTD) methods [10,15–17], finite element methods [11,13,29,30], and the monograph [12].

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Due to its efficiency and robustness, FDTD or Yee scheme, firstly introduced by Yee [31] in 1966, is still one of the most popular numerical methods in computational electromagnetics. The FDTD scheme uses central-difference approximations to the space and time partial derivatives at a fully staggered grid, and is second-order accurate in both time and space and easy to implement. For this aspect of theoretical study on the finite difference method for solving time-dependent Maxwell's equations, there are already excellent work in mathematical literature including FDTD scheme and related numerical methods, such as, the alternating direction implicit FDTD (ADI-FDTD) [20, 32], the energy conserved splitting FDTD (EC-S-FDTD) [5, 20], the splitting FDTD (S-FDTD) [2], the splitting multi-symplectic method [7] etc. Among the above-mentioned methods, the stability and error estimates in the L^2 norm have been studied by the energy method. In 2011, Gao and Zhang [4] were firstly studied the important stability and convergence analysis in the H^1 norm of the FDTD scheme with PEC boundary condition on uniform meshes, and extended the similar results to other relevant numerical methods [3, 18].

Recently, the theoretical analysis of the Yee scheme on non-uniform meshes have attracted much attention. The rigorous error analysis of the Yee scheme on non-uniform rectangular meshes can be traced back 1994 by Monk and Süli [19]. They used the special structure of local errors to prove that the Yee scheme still has second-order convergent on a non-uniform mesh although the local truncation error is only of the first order. Remis [21] studied the stability condition of the Yee scheme for solving the Maxwell's equations in lossless medium on non-uniform meshes, by the eigenvalues of the FDTD iteration matrix. In 2016, Li and Shields [14] extended Monk and Süli's technique to give the superconvergence analysis of Yee scheme for solving Maxwell's equations in metamaterials on non-uniform meshes, and extended to an implicit scheme [27]. In these work, the energy method was used to study the stability and error estimates in the L^2 norm. However, no results is available for the import stability and convergence analysis in H^1 norm of the Yee scheme for metamaterial Maxwell's equations on non-uniform rectangular meshes.

Encouraged by the nice properties of the Yee scheme for Maxwell's equations in simple media on uniform meshes [4], in this paper, we study the stability and convergence of the Yee scheme for metamaterial Maxwell's equations on non-uniform rectangular meshes by a new energy method. This new method is motivated by the new energy identities of metamaterial Maxwell's equations established in this paper and is different from the usual one in L^2 norm (cf. [14,27]). By making use of this new energy method, we prove that the Yee scheme with the PEC boundary condition on non-uniform rectangular meshes is stable in the discrete H^1 norm when the CFL condition is satisfied. Numerical results are also presented to confirm the theoretical analysis. Moreover, the superconvergence phenomena are proved for solving metamaterial Maxwell's equations by the Yee scheme on non-uniform rectangular and cubic meshes. To our best knowledge, this is the first result for the important stability analysis in the discrete H^1 norm of the Yee scheme for solving metamaterial Maxwell's equations on non-uniform rectangular meshes.

The rest of the paper is organized as follows. In Section 2, the new energy identities