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Interacting Stress Intensity Factors of Multiple Elliptical-Holes and Cracks Under Far-Field and Arbitrary Surface Stresses

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Abstract. Calculating interacting stress intensity factors (SIFs) of multiple elliptical-holes and cracks is very important for safety assessment, stop-hole optimization design and resource exploitation production in underground rock engineering, e.g., buried tunnels, deep mining, geothermal and shale oil/gas exploitation by hydraulic fracturing technology, where both geo-stresses and surface stresses are applied on buried tunnels, horizontal wells and natural cracks. However, current literatures are focused mainly on study of interacting SIFs of multiple elliptical-holes (or circular-holes) and cracks only under far-field stresses without consideration of arbitrary surface stresses. Recently, our group has proposed a new integral method to calculate interacting SIFs of multiple circular-holes and cracks subjected to far-filed and surface stresses. This new method will be developed to study the problem of multiple elliptical-hole and cracks subjected to both far-field and surface stresses. In this study, based on Cauchy integral theorem, the exact fundamental stress solutions of single elliptical-hole under arbitrarily concentrated surface normal and shear forces are derived to establish new integral equation formulations for calculating interacting SIFs of multiple elliptical-holes and cracks under both far-field and arbitrary surface stresses. The new method is proved to be valid by comparing our results of interacting SIFs with those obtained by Green's function method, displacement discontinuity method, singular integral equation method, pseudo-dislocations method and finite element method. Computational examples of one elliptical-hole and one crack in an infinite elastic body are given to analyze influence of loads and geometries on interacting SIFs. Research results show that when $\sigma_{xx}^{\infty} \geq \sigma_{yy}^{\infty}$, there appears a neutral crack orientation

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angle β_0 (without elliptical-hole's effect). Increasing $\sigma_{xx}^{\infty}/\sigma_{yy}^{\infty}$ and b/a (close to circular-hole) usually decreases β_0 of K_I and benefits to the layout of stop-holes. The surface compressive stresses applied onto elliptical-hole (n) and crack (p) have significant influence on interacting SIFs but almost no on β_0. Increasing n and p usually results in increase of interacting SIFs and facilitates crack propagation and fracture networks. The elliptical-hole orientation angle (α) and holed-cracked distance (t) have great influence on the interacting SIFs while have little effect on β_0. The present method is not only simple (without any singular parts), high-accurate (due to exact fundamental stress solutions) and wider applicable (under far-field stresses and arbitrarily distributed surface stress) than the common methods, but also has the potential for the anisotropic problem involving multiple holes and cracks.

AMS subject classifications: 74A10, 78B05

Key words: Interacting stress intensity factors, multiple elliptical-holes and cracks, far-field stresses, arbitrary surface stresses, integral equation method.

1 Introduction

In underground rock engineering such as buried tunnels, deep mining, geothermal and shale oil/gas exploitation by hydraulic fracturing technology, both the geo-stresses and surface stresses (e.g., seepage pressure) are applied onto the buried tunnels, horizontal wells and natural cracks [1–4]. In addition, buried tunnels are usually designed in elliptical cross-sections for the improving stability of tunnels [5,6] and horizontal wells actually presents the form of elliptical-holes due to the long-term action of gravity stresses [7, 8]. Under the external loads, the elliptical-holes have probability of facilitating or restraining initiation and propagation of natural cracks existing in the rock mass, since the different layouts of elliptical-holes can enlarge or reduce interacting stress intensity factors (SIFs). Consequently, calculating interacting SIFs of multiple elliptical-holes and cracks under complex loads plays an important role in safety assessment, stop-hole optimization design and resource exploitation production in underground rock engineering.

Up to now, many methods are available for calculating interacting SIFs of circular-holes and cracks under far-field loadings. For instance, Laurent series expansion method [9, 10] and Green's function method [11, 12] were used for one circular-hole and one crack, and boundary collection method [13], volume integral equation method [14], singular integral equation method [15] and boundary integral method [16, 17] were adopted for the problem of multiple circular-holes and cracks. Very few literatures are reported for surface stresses applied on the circular-holes and cracks, especially for arbitrary surface stresses. In addition, there are also some methods for calculating interacting SIFs of elliptical-holes and cracks. For example, perturbation method [18], singular integral equation method [19] and boundary element method [20] were applied for solving the problem of one elliptical-hole and one crack, one elliptical-hole and multiple cracks, multiple elliptical-holes and one crack, respectively. For more complicated