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s-Sequence-Covering Mappings on Metric Spaces

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Abstract. In this paper, we introduce and study s-sequence-covering mappings and 1-s-sequence-covering mappings, obtain some characterizations of s-sequence-covering and compact images of metric spaces, and prove that every s-sequence-covering and compact mapping in first-countable spaces is a 1-s-sequence-covering mapping.

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1 Introduction

Statistical convergence as a generalization of the usual notion of convergence was introduced by H. Fast [1] and H. Steinhaus [2]. There is not doubt that the study of statistical convergence and its various generalizations has become an active research area [3–8]. The original notion of statistical convergence was introduced for the real space $\mathbb{R}$. Generally speaking, this notion was extended in two directions. One is to discuss statistical convergence in more general spaces, for example, locally convex spaces [9], Banach spaces with the weak topologies [6,10,11], and topological spaces [5,7,8]. The other is to consider generalized notions defined by various limit processes, for example, A-statistical convergence [12], lacunary statistical convergence [13], and λ-statistical convergence [14]. Perhaps, a most general notion of statistical convergence is ideal (or filter) convergence [15, 16]. On the other hand, to find the internal characterizations of certain images of metric spaces is one of the central questions in general topology. F. Siwiec [17] introduced the concept of sequence-covering mappings. Thereafter, the research in this area has been well developed [18–22].

As we know, sequence-covering mappings, 1-sequence-covering mappings and sequentially quotient mappings are one of the most important tools to study certain images

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of metric spaces [19]. Recently, V. Renukadevi and B. Prakash defined two new sequence-covering mappings about statistical convergence as follows: Let $f:X\to Y$ be a mapping. The mapping f is said to be a *statistically sequence-covering mapping* [23], if for a given sequence $\{y_n\}_{n\in\mathbb{N}}$ with $y_n\to y$ in Y, there exists a sequence $\{x_n\}_{n\in\mathbb{N}}$ which statistically converges to a point $x\in f^{-1}(y)$ and each $x_n\in f^{-1}(y_n)$; the mapping f is said to be a *statistically sequentially quotient mapping* [24], if for a given sequence $y_n\to y$ in Y, there exists a sequence $x_k\to x\in f^{-1}(y)$ such that the sequence $\{f(x_k)\}_{k\in\mathbb{N}}$ is statistically dense in $\{y_n\}_{n\in\mathbb{N}}$. They discussed the relationship among sequence-covering mappings, statistically sequence-covering mappings and statistically sequentially quotient mappings, and studied their roles in the images of metric spaces.

Theorem 1.1 ([24]). *Let $f:X\to Y$ be a statistically sequentially quotient and boundary-compact map. If the space X is an open and compact-covering image of some metric space, then f is a 1-sequence-covering map.*

It is well known that we have the following result for the usual convergence.

Theorem 1.2 ([22]). *The following are equivalent for a topological space X:*
(1) *X is a sequence-covering and compact image of a metric space.*
(2) *X is a 1-sequence-covering and compact image of a metric space.*
(3) *X has a point-star network consisting of point-finite cs-covers.*
(4) *X has a point-star network consisting of point-finite sn-covers.*

We wonder if there are similar results for the case of statistical convergence? For this reason, this paper introduces and discusses s-sequence-covering mappings and 1-s-sequence-covering mappings. It is expected that s-sequence-covering mappings and 1-s-sequence-covering mappings shall also play an active role.

2 Preliminaries

In this paper, the set of all positive integers is denoted by $\mathbb{N}$, and the cardinality of the set B is denoted by $|B|$. The definition of statistical convergence of sequences is based on the notion of asymptotic density of a set $A\subset\mathbb{N}$.

Definition 2.1 ([25]). *Let $A\subset\mathbb{N}$ and $A(n)=\{k\in A: k\le n\}$ for each $n\in\mathbb{N}$. Then $\underline{\delta}(A)=\liminf_{n\to\infty}|A(n)|/n$ and $\overline{\delta}(A)=\limsup_{n\to\infty}|A(n)|/n$ are the lower and upper asymptotic density of the set A, respectively. If $\underline{\delta}(A)=\overline{\delta}(A)$, then $\delta(A)=\lim_{n\to\infty}|A(n)|/n$ is called the asymptotic density of A. A set $A\subset\mathbb{N}$ is said to be a statistically dense set if $\delta(A)=1$; a subsequence $\{x_{n_k}\}_{k\in\mathbb{N}}$ of a sequence $\{x_n\}_{n\in\mathbb{N}}$ is said to be statistically dense in $\{x_n\}_{n\in\mathbb{N}}$ if the set $\{n_k: k\in\mathbb{N}\}$ is statistically dense in $\mathbb{N}$.*

Definition 2.2 ([5]). *Let X be a topological space.*