

Hierarchical Absorbing Interface Conditions for Wave Propagation on Non-Uniform Meshes

Shuyang Dai^{1,2}, Zhiyuan Sun², Fengru Wang¹,
Jerry Zhijian Yang^{1,2,*} and Cheng Yuan³

¹ School of Mathematics and Statistics, Wuhan University, Wuhan 430072, P.R. China

² Computational Science Hubei Key Laboratory, Wuhan University, Wuhan 430072, P.R.China

³ Three Gorges Mathematical Research Center, China Three Gorges University, Yichang 443002, Hubei, China

Received 2 August 2021; Accepted (in revised version) 8 December 2021

Abstract. In this paper, we propose hierarchical absorbing interface conditions to solve the problem of wave propagation in domains with a non-uniform space discretization or grid size inhomogeneity using Padé Via Lanczos (PVL) method. The proposed interface conditions add an auxiliary variable in the wave system to eliminate the spurious reflection at the interface between regions with different mesh sizes. The auxiliary variable with proper boundary condition can suppress the spurious reflection by cancelling the boundary source term produced by the space inhomogeneity in variational perspective. The new hierarchical interface conditions with the help of PVL implementation can effectively reduce the degree of freedom in solving the wave propagation problem.

AMS subject classifications: 65K10, 65N22, 35L05

Key words: Wave equation, absorbing interface condition, spurious reflection, Padé via Lanczos.

1. Introduction

Numerical methods such as finite element method (FEM) and finite difference method (FDM) are widely used for simulating various physical phenomenon such as structural response of materials, evolution of phase transformation and wave propagation in medium. Many scientific discoveries and industry advances rely on the development of modern numerical techniques. In most numerical methods, there is an im-

*Corresponding author. *Email addresses:* shuyang_dai@whu.edu.cn (S. Dai), sun_zhiyuan@iapcm.ac.cn (Z. Sun), wangfr@whu.edu.cn (F. Wang), zjyang.math@whu.edu.cn (J. Z. Yang), yuancheng@whu.edu.cn (C. Yuan)

portant question on how to determine the numerical spatial resolution or discretization mesh size in the actual implementation of simulations. Although if increasing the numerical spatial resolution or equivalently decreasing the discretization mesh size could provide more information and solve the problems with a higher precision, however, the computational cost due to the unified high spatial resolution in the whole domain of calculation is consuming. Since in most realistic simulations, the information we are interested in is only bounded in a small region, and the whole computational domain is relatively large, it is important to develop an unified numerical framework that could couple different models with various spatial resolution or discretization mesh size [8, 9, 18, 19]. This numerical framework could increase the grid resolution locally where it is required physically relevant shorter length scale of problem under consideration and adopt coarse mesh size where it is the rest of computational domain, which means that grid inhomogeneity is an essential ingredient in modern large scale numerical simulations.

However, in the problem of wave propagation, the main challenge to use grid inhomogeneity in numerical simulations is the spurious reflection, which is a nonphysical reflection wave occurring when a wave passes through the interface between two regions with grid size inhomogeneity. When wave packet propagating through the numerical interface between domain with different grid sizes, the waves will split into transmitted and reflected components due to the artificial nonphysical interface. Actually, wave propagation in discrete system has been studied since the 17th century and has been described by Brillouin in Ref. [6]. A one-dimensional lattice of a point mass connected by springs has been considered as a model for wave propagation in crystals. Bazant in [2, 3, 7] found that when elastic wave pass through the different scale meshes, the spurious reflection may appear at the interface, and the effect of consistent and lumped mass models. The finite element approximations of nonlinear elastic waves were studied in Ref. [21]. In Ref [16], Jiang *et al.* studied the spurious wave reflection at the interface of different physical properties in finite element wave solution. The optimal reduction for wave propagation problems in the numerical dispersion relationship using two-dimensional elements is discussed in Ref. [13]. Kulkarni *et al.* [14, 15] has talked about this topic in the theory of peridynamic, including the wave propagation in a peridynamic bar with nonuniform discretization and the spurious wave reflection at the interface between peridynamics and finite element regions.

One related work in the multi-scale modeling and computation is the coupling of the molecular dynamics model and the continuum model. An important work in this region is the heterogeneous multiscale method (HMM) developed by E *et al.* in Ref. [9, 17, 20, 27], which is based on the widely used domain decomposition method in finite elements and the Cauchy-Born rule. Another interesting method is the bridging scale method (BSM) developed by Liu *et al.*, they developed the mathematical framework of the bridging scale method and use a projection process to couple atomistic and continuum simulations using a bridging scale decomposition [26, 28]. Another approach to deal with the interface in PDEs is local time stepping method, when the space is discretized by adaptive mesh. Osher and Sanders studied the 1-D case of nu-