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EXISTENCE OF PERIODIC SOLUTION FOR A KIND OF THIRD-ORDER GENERALIZED NEUTRAL FUNCTIONAL DIFFERENTIAL EQUATION WITH VARIABLE PARAMETER*

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Abstract

In this paper, we investigate a third-order generalized neutral functional differential equation with variable parameter. Based on Mawhin's coincidence degree theory and some analysis skills, we obtain sufficient conditions for the existence of periodic solution for the equation. An example is also provided.

Keywords existence of periodic solution; third-order neutral functional differential equation; variable parameter; Mawhin's continuation theorem; coincidence degree

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1 Introduction

Neutral differential equations are widely used in many fields including biology, chemistry, physics, medicine, population dynamics, mechanics, economics, and so on (see [6,8,10,27]). For example, in population dynamics, since a growing population consumes more (or less) food than a matured one, depending on individual species, this leads to neutral equations [10]. These equations also arise in classical cobweb models in economics where current demand depends on price, but supply depends on the previous periodic [6]. In recent years, the problem of the existence of periodic solutions for neutral differential equations has been extensively studied in the literature. We refer the reader to [1-5,11-14,17-19,21-24] and the references cited therein for more details.

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In this paper, we consider the generalized neutral functional differential equation with variable parameter

$$\frac{\mathrm{d}^3}{\mathrm{d}t^3}\big(x(t)-c(t)x(t-\delta(t))\big)+f\big(t,\ddot{x}(t)\big)+g\big(t,\dot{x}(t)\big)+h\big(t,x\big(t-\tau(t)\big)\big)=e(t), \quad (1)$$

where $|c(t)|\neq 1,\ c,\delta\in C^2(\mathbb{R},\mathbb{R})$ and c,δ are ω-periodic functions for some $\omega>0$, $\tau,e\in C[0,\omega]$ and $\int_0^\omega e(t)\mathrm{d}t=0$; f,g and h are continuous functions defined on $\mathbb{R}^2$ and periodic in t with $f(t,\cdot)=f(t+\omega,\cdot)$, $g(t,\cdot)=g(t+\omega,\cdot)$, $h(t,\cdot)=h(t+\omega,\cdot)$, and $f(t,0)=g(t,0)=0$.

In recent years, when $c(t)$ is a constant c or $\delta(t)$ is a constant δ or both of them are constants, many researchers have extensively studied such types of neutral functional differential equations. We refer the reader [9,15-17,20,26] and their references therein. But the work to study the existence of periodic solutions for neutral functional differential equations with variable parameter has rarely appeared. There are two reasons for this. The first reason is that the criterion of L-compact of nonlinear operator N on the set $\overline{\Omega}$ is difficult to establish when $c(t)$ is not a constant. The second reason is that the linear operator $A: C_T\to C_T$, $[Ax](t)=x(t)-c(t)x(t-\tau)$, for all $t\in[0,T]$, has continuous inverse A^{-1}, which is far away from the answer.

For example, Du et al. [5] investigated the second-order neutral equation

$$\big(x(t)-c(t)x(t-\delta)\big)''+f\big(x(t)\big)x'(t)+g\big(x\big(t-\gamma(t)\big)\big)=e(t), \quad (2)$$

by using Mawhin's continuous theorem, the authors obtained the existence of periodic solution for (2).

Afterwards, in [19], Ren et al. considered the following neutral differential equation with deviating arguments:

$$\big(x(t)-cx\big(t-\delta(t)\big)\big)''=f(t,x'(t))+g\big(t,x\big(t-\tau(t)\big)\big)+e(t),$$

by the continuation theorem and some analysis techniques, some new results on the existence of periodic solutions were obtained.

Recently, Xin and Zhao [25] studied the neutral equation with variable delay

$$\big(x(t)-c(t)x\big(t-\delta(t)\big)\big)''+f(t,x'(t))+g\big(t,x\big(t-\tau(t)\big)\big)=e(t), \quad (3)$$

by coincidence degree theory and some analysis skills, the authors obtained sufficient conditions for the existence of periodic solution for (3).

Motivated by [5,19,25], in this paper, we consider the generalized neutral equation (1). Notice that here the neutral operator A is a natural generalization of the familiar operator $A_1=x(t)-cx(t-\delta)$, $A_2=x(t)-c(t)x(t-\delta)$, $A_3=x(t)-cx(t-\delta(t))$. But A possesses a more complicated nonlinearity than A_i, $i=1,2,3$. For example, the neutral operator A_1 is homogeneous in the following sense $\frac{\mathrm{d}}{\mathrm{d}t}(A_1x)(t)=(A_1\dot{x})(t)$,