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AN ITERATIVE METHOD FOR SPLIT VARIATIONAL INCLUSION PROBLEM AND FIXED POINT PROBLEM FOR A FAMILY OF GENERALIZED ASYMPTOTICALLY NONEXPANSIVE SEMIGROUP*†

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Abstract

In this paper, strong convergence of an iterative sequence is proved, which computes an approximate solution of the set of solutions of split variational inclusion problem, the set of fixed points of a nonexpansive mapping and the set of common fixed points of a family of generalized asymptotically nonexpansive semigroup. Results obtained in this paper extend and unify the previously known results in the previous literatures.

Keywords split variational inclusion problem; strong convergence theorem; fixed point problems; generalized asymptotically nonexpansive semigroup

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1 Introduction and Preliminaries

Let C and Q be nonempty closed convex subsets of real Hilbert spaces H_1 and H_2, respectively. The split feasibility problem (SFP) is formulated as:

$$\text{finding an } x^* \in C \text{ such that } Ax^* \in Q, \tag{1.1}$$

where $A : H_1 \to H_2$ is a bounded linear operator. In 1994, Censor and Elfving [1] first introduced the SFP in finite-dimensional Hilbert spaces modeling inverse problems. It has been found that the SFP can also be used in various disciplines such as image restoration, computer tomograph and radiation therapy treatment

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planing [2-4]. The SFP in an infinite dimensional real Hilbert space can be found in [4-8].

A popular algorithm to be used to solves the SFP (1.1) is due to Byrne's CQ-algorithm [5]:

$$x_{k+1} = P_C(I - \beta_k A^*(I - P_Q)A)x_k, \quad k \geq 1, \tag{1.2}$$

where $\beta_k \in (0, 2/\lambda)$ with λ being the spectral radius of the operator A^*A.

Let $A : H_1 \to H_2$ be a bounded linear operator. Let $B : H_1 \to H_1$ and $K : H_2 \to H_2$ be maximal monotone mappings. The "so-called" classical split variational inclusion problem (CSVIP) is:

$$\text{to find an } x^* \in H_1 \text{ such that } 0 \in B(x^*) \text{ and } 0 \in K(Ax^*), \tag{1.3}$$

which was introduced by Moudafi [9]. We denote the solution set of CSVIP (1.3) by Γ. In [9], Moudafi proved that the following iteration process

$$x_{k+1} = J_\lambda^B(x_k + \gamma A^*(J_\lambda^K - I)Ax_k), \quad k \geq 1 \tag{1.4}$$

converges weakly to a solution of problem (1.3), where λ and γ are given positive numbers.

In [10], K.R. Kazmi and S.H. Rizvi analyzed the following iterative method for approximating a common solution of CSVIP

$$\begin{cases} u_n = J_\lambda^B(x_n + \gamma A^*(J_\lambda^K - I)Ax_n), \quad n \geq 1, \\ x_{n+1} = a_n f(x_n) + (1 - a_n)Su_n, \end{cases}$$

where $f : H_1 \to H_1$ is a contraction and $S : H_1 \to H_1$ is a nonexpansive mapping. Under certain conditions, the strong convergence of sequence $\{x_n\}$ is proved.

Let $F : C \to C$ be a nonlinear mapping. The variational inequality problem is:

$$\text{to find an } x \in C \text{ such that } \langle Fx, y - x \rangle \geq 0, \text{ for any } y \in C. \tag{1.5}$$

Now, we recall the well-known concepts and results.

Definition 1.1 A mapping $T : C \to C$ is said to be contractive if there exists a constant $\alpha \in (0, 1)$ such that

$$\|Tx - Ty\| \leq \alpha\|x - y\|.$$

Definition 1.2 A mapping $T : C \to C$ is said to be weakly contractive if there exists a nondecreasing function $\psi : [0, \infty) \to [0, \infty)$ satisfying $\psi(t) = 0$ if and only if $t = 0$ and

$$\|Tx - Ty\| \leq \|x - y\| - \psi(\|x - y\|).$$

It is known that the class of weakly contractive mappings contain properly the class of contractive ones, see [11,12].