Ann. of Appl. Math.
32:1(2016), 69-78

CYCLES EMBEDDING ON FOLDED HYPERCUBES WITH FAULTY NODES*†

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Abstract

Let FF_v be the set of faulty nodes in an n-dimensional folded hypercube FQ_n with $|FF_v| \le n-1$ and all faulty vertices are not adjacent to the same vertex. In this paper, we show that if $n \ge 4$, then every edge of $FQ_n - FF_v$ lies on a fault-free cycle of every even length from 6 to $2^n - 2|FF_v|$.

Keywords folded hypercube; interconnection network; fault-tolerant; path

2000 Mathematics Subject Classification 68M15; 68M10

1 Introduction

The n-dimensional hypercube Q_n (or n-cube) is one of the most important topology of networks due to its excellent properties such as regularity, recursive structure, small diameter, vertex and edge transitive and relatively short mean distance [1]. In order to improve the performance of hypercube, the folded hypercube FQ_n has been proposed [2].

Since a large-scale hypercube network fails in any component, it's desirable that the rest of the network continue to operate in spite of the failure. This leads to the graph-embedding problem with faulty edges and/or vertices. This problem has received much attention (see [3-10]).

The problem of embedding paths in an n-dimensional hypercube and folded hypercube has been well studied. Tsai [3] showed that for any subset F_v of $V(Q_n)$ with $|F_v| \le n-2$, every edge of $Q_n - F_v$ lies on a cycle of every even length from 4 to $2^n - 2|F_v|$ inclusive. Tsai [4] also showed that for any subset F_v of $V(Q_n)$ with $|F_v| \le n-1$ and all faulty vertices are not adjacent to the same vertex, every edge of $Q_n - F_v$ lies on a cycle of every even length from 6 to $2^n - 2|F_v|$ inclusive. Hsieh

*This project was supported by NSFC (11371162) and NSFC(11171129) and HuBei (T201103).

†Manuscript received October 10, 2015

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and Shen [5] proved that every edge of $Q_n - F_v - F_e$ lies on a cycle of every even length from 4 to $2^n - 2|F_v|$ even if $|F_v| + |F_e| \leq n - 2$, where $n \geq 3$.

Let FF_v and FF_e denote the set of faulty nodes and faulty edges of FQ_n respectively. Hsieh, Kuo and Huang [6] proved that if the folded hypercube FQ_n has just only one fault node, then FQ_n contains cycles of every even length from 4 to $2^n - 2$ if $n \geq 3$, and cycles of every odd length from $n+1$ to $2^n - 1$ when n is even, $n \geq 2$. Ma, Xu and Du [7] further demonstrated that $FQ_n - FF_e$ $(n \geq 3)$ with $|FF_e| \leq 2n - 3$ contains a fault-free cycle passing through all nodes if each vertex is incident with at least two fault-free edges. Kuo and Hsieh [8] improved the conclusion of [7] and proved that $FQ_n - FF_e$ with $|FF_e| = 2n - 3$ contains a fault-free cycle of every even length from 4 to 2^n. Xu, Ma and Du [9] further showed that every fault-free edge of $FQ_n - FF_e$ lies on a fault-free cycle of every even length from 4 to 2^n and every odd length from $n+1$ to $2^n - 1$ if n is even, where $|FF_e| \leq n - 1$. Then Cheng, Hao and Feng [10] proved that every fault-free edge of $FQ_n - FF_v$ lies on a fault-free cycle of every even length from 4 to $2^n - 2|FF_v|$ and every odd length from $n + 1$ to $2^n - 2|FF_v| - 1$ if n is even, where $|FF_v| \leq n - 2$.

In this paper, under the conditional $|FF_v| \leq n - 1$ and all faulty vertices are not adjacent to the same vertex, we show that if $n \geq 4$, then every edge of $FQ_n - FF_v$ lies on a fault-free cycle of every even length from 6 to $2^n - 2|FF_v|$.

2 Preliminaries

Please see [1] for graph-theoretical terminology and notation is not defined here. A network is usually modeled by a simple connected graph $G = (V, E)$, where $V = V(G)$ (or $E = E(G)$) is the set of vertices (or edges) of G. We define the vertex x to be a neighbor of y if $xy \in E(G)$. A graph G is bipartite if X, Y are two disjoint subsets of $V(G)$ such that $E(G) = \{xy|\, x \in X, y \in Y\}$. A graph $P = (u_1, u_2, \cdots, u_k)$ is called a path if the vertices $u_1, u_2, \cdots, u_k$ are distinct and any two consecutive vertices u_i and u_{i+1} are adjacent. u_1 and u_k are called the end-vertices of P. If $u_1 = u_k$, the path $P(u_1, u_k)$ is called a cycle (denoted by C). The length of a path P (a cycle C), denoted by $l(P)$ (or $l(C)$), is the number of edges in P (or C). In general, the distance of two vertices x, y is the length of the shortest (x, y)-path.

The n-dimensional hypercube Q_n (or, n-cube) can be represented as an undirected graph with 2^n vertices. Every vertex $x \in Q_n$ is labeled as a binary string $x_1 x_2 \cdots x_n$ of length n from $00 \cdots 0$ to $11 \cdots 1$. Two vertices u and v are adjacent if their binary strings differ in exactly one bit. For convenience, we call $e \in E$ an edge of dimension i if its end-vertices strings differ in ith-bit. In the rest of this paper, we denote $x^i = x_1 x_2 \cdots \overline{x_i} \cdots x_n$, where $\overline{x_i} = 1 - x_i$, $x_i = 0, 1$. The Hamming