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ON ALMOST AUTOMORPHIC SOLUTIONS OF THIRD-ORDER NEUTRAL DELAY-DIFFERENTIAL EQUATIONS WITH PIECEWISE CONSTANT ARGUMENT*†

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Abstract

We present some conditions for the existence and uniqueness of almost automorphic solutions of third order neutral delay-differential equations with piecewise constant of the form

$$(x(t)+px(t-1))'''=a_0x([t])+a_1x([t-1])+f(t),$$

where $[\cdot]$ is the greatest integer function, p, a_0 and a_1 are nonzero constants, and $f(t)$ is almost automorphic.

Keywords almost automorphic solutions; neutral delay equation; piecewise constant argument

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1 Introduction

In this paper we study certain functional differential equations of neutral delay type with piecewise constant argument of the form

$$(x(t)+px(t-1))'''=a_0x([t])+a_1x([t-1])+f(t), \tag{1}$$

here $[\cdot]$ is the greatest integer function, p, a_0 and a_1 are nonzero constants, and $f(t)$ is almost automorphic.

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By a solution $x(t)$ of (1) on $\mathbb{R}$ we mean a function continuous on $\mathbb{R}$, satisfying (1) for all $t \in \mathbb{R}$, $t \neq n \in \mathbb{Z}$, and such that the one sided third derivatives of $x(t) + px(t-1)$ exist at $n \in \mathbb{Z}$.

The concept of almost automorphic functions is more general than that of almost periodic functions, which were introduced by S. Bochner [1,2], for more details about this topics we refer to [3,4,6-9] and references therein.

Differential equations with piecewise constant argument (EPCA), which were firstly considered by Cooke and Wiener [11], and Shah and Wiener [12], describe the hybrid of continuous and discrete dynamical systems, which combine the properties of both differential equations and difference equations and have applications in certain biomedical models in the works of Busenberg and Cooke in [13]. Therefore, there are many papers concerning the differential equations with piecewise constant argument (see e.g. [14-20] and references therein). However, there are only a few works on the almost automorphy of solutions of EPCAs. To the best of our knowledge, only Minh et al [21] in 2006, Dimbour [22] in 2011 and Li [23] in 2013 studied in this line. They give sufficient conditions for the almost automorphy of bounded solutions of differential equation EPCAs.

Motivated by the above works, in this paper we investigate the existence of almost automorphy solutions of equation (1). The paper is organized as follows. In Section 2, some notation, preliminary definitions and lemmas are presented. The man result and its proofs is put in Sections 3.

2 Preliminary Definitions and Lemmas

Throughout this paper, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}$ and $\mathbb{C}$ denote the sets of natural numbers, integers, real and complex numbers, respectively. $l^\infty(\mathbb{R})$ denotes the space of all bounded (two-sided) sequences $x : \mathbb{Z} \to \mathbb{R}$ with sup-norm. We always denote by $|\cdot|$ the Euclidean norm in $\mathbb{R}^k$ or $\mathbb{C}^k$, and by $BC(\mathbb{R}, \mathbb{R})$ the space of bounded continuous functions $u : \mathbb{R} \to \mathbb{R}$.

Definition 2.1 A continuous function $f : \mathbb{R} \to \mathbb{R}$ is called almost automorphic if for every sequence of real numbers $(s'_n)_{n\in\mathbb{N}}$, there exists a subsequence $(s_n)_{n\in\mathbb{N}}$ such that

$$g(t) = \lim_{n\to\infty} f(t + s_n)$$

is well defined for each $t \in \mathbb{R}$ and

$$f(t) = \lim_{n\to\infty} g(t - s_n)$$

for each $t \in \mathbb{R}$. The collection of such functions is denoted by $\mathcal{AA}(\mathbb{R})$.

It is clear that the function g in Definition 2.1 is bounded and measurable.