Journal of Nonlinear Modeling and Analysis http://jnma.ca; http://jnma.ijournal.cn
Volume 4, Number 1, March 2022, 18–41 DOI:10.12150/jnma.2022.18

Transversal Heteroclinic Bifurcation in Hybrid Systems with Application to Linked Rocking Blocks*

Mi Zhou[1] and Zhengdong Du[1,†]

Abstract In this paper, we study heteroclinic bifurcation and the appearance of chaos in time-perturbed piecewise smooth hybrid systems with discontinuities on finitely many switching manifolds. The unperturbed system has a heteroclinic orbit connecting hyperbolic saddles of the unperturbed system that crosses every switching manifold transversally, possibly multiple times. By applying a functional analytical method, we obtain a set of Melnikov functions whose zeros correspond to the occurrence of chaos of the system. As an application, we present an example of quasiperiodically excited piecewise smooth system with impacts formed by two linked rocking blocks.

Keywords Melnikov method, Hybrid system, Heteroclinic bifurcation, Chaos, Linked rocking blocks.

MSC(2010) 34C23, 34C37, 37C29.

1. Introduction

It is very important and interesting in the theory of dynamical systems to investigate the occurrence of chaos. Common routes to chaos for smooth systems include period-doubling, intermittency, torus bifurcation and homoclinic bifurcation [26,42]. In particular, for a periodically excited smooth system with a homoclinic orbit, the perturbed stable and unstable manifolds intersect transversally under some conditions, which implies the existence of Smale horseshoe chaos via Smale-Birkhoff Homoclinic Theorem. The Melnikov method is a powerful analytical tool that can be used to determine whether transversal homoclinic intersection occurs [19,25,26,40].

In recent years, with the development of science and technology, there have been lots of works devoted to the study of bifurcation phenomena in piecewise smooth (PWS) dynamical systems [9, 16, 19, 38, 39, 46]. This is because many problems from real applications in fields such as mechanics, electrical engineering and control theory are modelled by PWS systems. For such systems, a typical route to chaos is through discontinuity-induced bifurcations, such as grazing, sliding, border-collision and chattering [2, 9, 13, 16, 19, 20, 32, 33, 39, 46].

In [15, 43], Chow, Rand and Shaw studied homoclinic bifurcations for a class of periodically excited linear inverted pendulum. Their numerical results suggest

† the corresponding author.
Email address: zhoum_aa@sina.com (M. Zhou), zdu1985@qq.com (Z. Du)
[1] Department of Mathematics, Sichuan University, Chengdu, Sichuan 610064, China

* The authors were supported by Natural Science Foundation of China (No. 11971019).

that homoclinic bifurcation can also lead to chaotic motions for PWS systems. In the last decades, many efforts have been made on extending the Melnikov method established for smooth systems to PWS systems. To mention only a few of them, see [2, 3, 14, 18–20, 22, 31, 36, 45]. It is assumed in these works that the unperturbed homoclinic or periodic orbit intersects the switching manifold transversally. In [1, 4, 5, 7, 19], Battelli, Fečkan, Awrejcewicz et al. extended the Melnikov method to sliding homoclinic bifurcation of general n-dimensional PWS systems. Grazing homoclinic bifurcation for a periodically excited nonlinear inverted pendulum was also studied in [17]. Calamai, Franca and Pospíšil [12, 21] investigated homoclinic bifurcations in PWS systems with the critical point lies on the switching manifold. In particular, they proved that in this case, the existence of a transversal homoclinic point does not imply chaos. In [4–8, 19], by using a functional analytical method, Battelli and Fečkan proved rigorously that if a certain Melnikov function has a simple zero, then under some recurrent conditions, a time-perturbed PWS system with transversal or sliding homoclinic orbit behaves chaotically in the sense that the system has a Smale-like horseshoe.

In 1988, Bertozzi extended the Smale-Birkhoff Homoclinic Theorem and the Melnikov method, so they are applicable to heteroclinic bifurcations for smooth systems [10]. It is natural to ask if the transversal intersections of the perturbed stable and unstable manifolds of a heteroclinic orbit of PWS systems result in chaotic motions. Unfortunately, the Heteroclinic Theorem of Berttozzi [10] requires the corresponding Poincaré map to be differentiable. Thus, it cannot be applied to PWS systems, because this condition is not satisfied by most of the PWS systems. Nevertheless, the study of heteroclinic bifurcations in time-perturbed PWS systems has attracted increasing attention. Heteroclinic bifurcations for models of periodically excited slender rigid blocks were studied in the works of Bruhn and Koch [11], Hogan [28], Lenci and Rega in [34]. In [23], Granados, Hogan and Seara presented the Melnikov method for heteroclinic and subharmonic bifurcations in a periodically excited piecewise planar Hamiltonian system with two zones. The Melnikov method for heteroclinic bifurcations of a planar PWS system with impacts and of a general planar PWS system with finitely many zones were developed in [35] and [44] respectively. Although it is not rigorously proved, numerical simulations on concrete examples given in these works suggest that chaotic behavior can be resulted from heteroclinic bifurcations in PWS systems.

Recently, by applying the aforementioned functional analytic method developed by Battelli and Fečkan in [4–8,19], Li and Du [37] studied the appearance of chaos in time-perturbed n-dimensional PWS systems with heteroclinic orbit. They derived a set of Melnikov type functions whose zeros correspond to the occurrence of chaos of the system. To reduce the complexity, they assumed that the switching manifolds are supersurfaces intersecting at a connected $(n-2)$-dimensional submanifold, the unperturbed system has a hyperbolic saddle in each subregion and a heteroclinic orbit connecting those saddles that crosses every switching manifold transversally exactly once. However, in real applications, discontinuities of a PWS system may occur on more complicated sets and impacts may occur, when the flow of the system reaches the switching manifolds. Thus, it is necessary to extend the results obtained in [37] to systems with other types of switching manifolds and other types of PWS systems, for example, systems with impacts considered in [23, 35].

The aim of this paper is to extend the results of [37] to more general systems, namely n-dimensional time-perturbed PWS hybrid systems. We assume that the