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Global Dynamics of a Diffusive Leslie-Gower Predator-prey Model with Fear Effect*

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Abstract A diffusive Leslie-Gower predator-prey model with fear effect is considered in this paper. For the kinetic system, we show that the unique positive equilibrium is globally asymptotically stable. Moreover, we find that high levels of fear could decrease the population densities of both prey and predator in a long time. For the diffusive model, we obtain the similar results under certain conditions.

Keywords Leslie-Gower predator-prey model, Fear effect, Global stability.

MSC(2010) 35K57, 37N25, 92D25.

1. Introduction

There are extensive models to describe the interaction between predator and prey, and one of the classical models takes the following form:

$$\begin{cases} \dfrac{\partial u}{\partial t} = d_1\Delta u + ru - \alpha u^2 - \phi(u)v, & x \in \Omega,\ t > 0, \\ \dfrac{\partial v}{\partial t} = d_2\Delta v - mv + c\phi(u)v, & x \in \Omega,\ t > 0, \\ \partial_n u = \partial_n v = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x) \geq (\not\equiv) 0,\ v(x,0) = v_0(x) \geq (\not\equiv) 0, & x \in \Omega. \end{cases} \tag{1.1}$$

Here, $u(x,t)$ and $v(x,t)$ are the densities of the prey and predator at location x and time t respectively; Ω is a bounded domain in $\mathbb{R}^N$ with a smooth boundary $\partial\Omega$; n is the outward unit normal vector on $\partial\Omega$; $d_1, d_2 > 0$ are the diffusion coefficients of the prey and predator, respectively; $r > 0$ is the intrinsic growth rate of the prey; $\alpha > 0$ represents the intraspecific competition of the prey; $m > 0$ is the death rate of the predator; $c > 0$ is the conversion rate; $\phi(u)$ denotes the predator functional response to the prey density. The predator functional responses are generally classified into four Holling types: I-IV [8]. For the Holling type II predator functional response, there exist extensive results for the kinetic system of model (1.1), such as the global stability of the positive equilibrium and the existence and uniqueness of a limit cycle, see [3, 9, 11]. Yi et al. [32] considered the steady

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state and Hopf bifurcations for model (1.1) with Holling type II predator functional response, and see also [22] for the nonexistence of nonconstant positive steady states. Dynamics of model (1.1) with other Holling type predator functional responses could be found in [1, 17, 23–26, 30, 35] and references therein.

It has been found recently that the fear of the predator could reduce the birth rate of the prey, see [6, 7, 12, 27, 28, 34] and references therein. To model this fear effect, Wang et al. [28] first introduced a predator-dependent growth rate function for the prey. Actually, they proposed the following predator-prey model:

$$\begin{cases} \dfrac{du}{dt} = rf(k,v)u - du - \alpha u^2 - \phi(u)v, & t > 0, \\ \dfrac{dv}{dt} = -mv + c\phi(u)v, & t > 0, \\ u(0) = u_0 > 0,\ v(0) = v_0 > 0, \end{cases} \tag{1.2}$$

where $f(k,v)$ represents the effect of fear, $k > 0$ reflects the level of fear, and $f(k,v)$ satisfies the following assumption:

(**A**) $f(k,v)$ is smooth, $f(0,v) = f(k,0) = 1$, $\lim_{k\to\infty} f(k,v) = 0$ and $\dfrac{\partial f(k,v)}{\partial k} < 0$ for $v > 0$, and $\lim_{v\to\infty} f(k,v) = 0$ and $\dfrac{\partial f(k,v)}{\partial v} < 0$ for $k > 0$.

It was showed in [28] that, for model (1.2) with the Holling type II predator functional response, high levels of fear can stabilize the positive steady state, and low levels of fear can induce multiple limit cycles via subcritical Hopf bifurcations. Moreover, the corresponding PDE model of (1.2) with the predator-taxis were investigated in [29].

Another classical predator-prey model is the following Leslie-Gower predator-prey model proposed by Leslie and Gower [14, 15]:

$$\begin{cases} \dfrac{\partial u}{\partial t} = d_1\Delta u + ru - \alpha u^2 - \beta uv, & x \in \Omega,\ t > 0, \\ \dfrac{\partial v}{\partial t} = d_2\Delta v + \lambda v\left(1 - \dfrac{v}{u}\right), & x \in \Omega,\ t > 0, \\ \partial_n u = \partial_n v = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x) > 0,\ v(x,0) = v_0(x) \geq (\not\equiv) 0, & x \in \partial\Omega. \end{cases} \tag{1.3}$$

Here, the carry capacity of the predator depends on the density of the prey, and parameters d_1, d_2, r, α, β and λ are all positive constants. For the kinetic system of model (1.3), Hsu [10] obtained that the unique positive equilibrium is globally asymptotically stable, which attracts all the positive solutions. For the diffusive case, Du and Hsu [4] found that if $\alpha/\beta > s_0$, where $s_0 \in (\frac{1}{5}, \frac{1}{4})$. Then, the unique positive constant equilibrium is globally asymptotically stable. Moreover, the dynamics of delayed diffusive Leslie-Gower predator-prey models were analyzed in [2, 5, 31, 33] and references therein, see also [13, 16] for the dynamics of the Leslie-Gower predator-prey model with Allee effect.

In this paper, we revisit model (1.3) with fear effect as in model (1.2). Following