Journal of Nonlinear Modeling and Analysis
Volume 4, Number 2, June 2022, 245–276
http://jnma.ca; http://jnma.ijournal.cn
DOI:10.12150/jnma.2022.245

A Family of Variable Step-size Meshes Fourth-order Compact Numerical Scheme for (2+1)-dimensions Burger's-Huxley, Burger's-Fisher and Convection-diffusion Equations

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Abstract Existing numerical schemes, maybe high-order accurate, are obtained on uniformly spaced meshes and challenges to achieve high accuracy in the presence of singular perturbation parameter, and nonlinearity remains left on nonuniformly spaced meshes. A new scheme is proposed for nonlinear 2D parabolic partial differential equations (PDEs) that attain fourth-order accuracy in xy-space and second-order exact in the temporal direction for uniform and nonuniform mesh step-size. The method proclaims a compact character using nine-point single-cell finite-difference discretization on a nonuniformly spaced spatial mesh point. A description of splitting compact operator form to the convection-dominated equation is obtained for implementing alternating direction implicit scheme. The procedure is examined for consistency and stability. The scheme is applied to linear and nonlinear 2D parabolic equations: convection-diffusion equations, Burger's-Huxley, Burger's-Fisher and coupled Burger's equation. The technique yields the tridiagonal matrix and computed by the Thomas algorithm. Numerical simulations with linear and nonlinear problems corroborate the theoretical outcome.

Keywords Nonlinear parabolic partial differential equations (PDEs), Two-dimensions Burger's-Huxley equation, Boussinesq equation, Convection-diffusion equation, Compact-scheme, Stability, Errors and numerical order.

MSC(2010) 65M12, 65M06.

1. Introduction

The parabolic partial differential equations (PDEs) appear to model ample physical phenomena in fluid flow, acoustic waves, mass transfer, groundwater, air pollution, chemical separation, shock wave, logistic population growth and nuclear reactor theory [4, 27, 28, 41]. In the past, Cole [7] described the analytic solution method for one-dimension Burger's equation, appearing in a turbulence model and weak

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nonstationary shock wave. Subsequently, a two-dimensional form of Burger's equation was reported by Fletcher [14] and Bhatt [2]. There are two essential variants to Burger's equation, namely Burger's Fisher and Burger's Huxley model, that occupy a prominent place in mathematical physics and many application areas. The 2D Burger's Fisher equation occurs in turbulence gas dynamics and plasma physics. It has diffusion transport and reaction behavior from the Fisher equation and convective phenomenon from the Burgers equation. The two-dimensional Burger's Fisher equation takes the form

$$\epsilon\left(\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}\right)=\frac{\partial W}{\partial t}+aW^n\left(\frac{\partial W}{\partial x}+\frac{\partial W}{\partial y}\right)+bW\left(W^n-1\right), \tag{1.1}$$

and Burger's Huxley equation appearing in nerve propagation, population genetics is represented by

$$\epsilon\left(\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}\right)=\frac{\partial W}{\partial t}+aW^n\left(\frac{\partial W}{\partial x}+\frac{\partial W}{\partial y}\right)+W\left(W^n-1\right)\left(W^n-\sigma\right). \tag{1.2}$$

The 2D Burger's Fisher equation (1.1) and Burger's Huxley equations (1.2) can be represented in the following mildly nonlinear equation

$$\epsilon\left(\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}\right)=\psi\left(x,y,t,W,\frac{\partial W}{\partial x},\frac{\partial W}{\partial y},\frac{\partial W}{\partial t}\right),(x,y,t)\in\Omega\times(0,T),\Omega\in\mathbf{R^2} \tag{1.3}$$

along with initial and boundary data

$$W(x,y,0)=\phi(x,y), \tag{1.4}$$
$$W(0,y,t)=G_1(y,t),W(1,y,t)=G_2(y,t), \tag{1.5}$$
$$W(0,x,t)=F_1(x,t),W(x,1,t)=F_2(y,t). \tag{1.6}$$

The nonlinearity of first-order partial derivatives appearing in the parabolic PDEs (1.3) makes it difficult to determine the analytic solution. Various computational techniques, namely finite-difference, homotopy perturbation, finite-element, adomain decomposition, finite-volume and spectral methods, were presented to analyze mildly nonlinear convection dominated diffusion models approximately. The solution schemes using compact discretizations in the finite-difference have been described in the past [17, 18, 33]. Noye and Tan [37] presented a stable third-order accurate in space and second-order accurate in time, implicit weighted compact discretization to the 2D constant-coefficient linear advection-diffusion. A high-order uniform meshes compact formulation for solving time-dependent linear convection dominated diffusion using the boundary value method was presented by Dehghan and Mohebbi [8]. A monotone finite difference discretization to singularly perturbed parabolic PDEs and analysis of their uniform convergence was described by Clavero and Jorge [6]. A lattice Boltzmann model to compute 2D Burger's equation and its stability was discussed by Duan and Liu [9]. The analysis of a high-order difference scheme for convection dominated diffusion problems has been recently obtained in [1, 36, 38]. Recently, the one-dimensional Burgers equation has been described by Jiwari [23]. It is long-familiar that the analytic solution of singularly perturbed nonlinear parabolic PDEs for an arbitrary selection of the function ψ is not possible, and numerical solution by classical finite-difference or finite-element method may portray multiple characters or rapid change of solution in a specific