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Analysis of the Dynamics of a Predator-prey Model with Holling Functional Response

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Abstract A diffusive predator-prey system with Holling functional response is considered. Firstly, existence of positive equilibrium of this reaction diffusion model under Neumann boundary condition is obtained. Meanwhile, the existence conditions for Turing instability and Hopf bifurcations of a system with Holling II functional response are established. Next, the existence of the hydra effect is demonstrated, when the system is undergoing non-homogeneous steady-state solutions. Finally, numerical simulations are illustrated to support our theory results.

Keywords Predator-prey model, Turing instability, Hopf bifurcation, Hydra effect.

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1. Introduction

Dynamics of predator-prey model is one of important subjects in mathematical ecology, and some important results have been studied and derived by many researchers [8, 10, 15, 18, 19, 21–23]. The earliest population can be traced back to 1798, the Malthus model proposed by T. R. Malthus. In 1838, the Dutch biologist P. Verhulst introduced the largest population that the natural environment could withstand on the basis of this model, and proposed the famous Logistic model. In 1948, P. H. Leslie and J. C. Gower extended the Logistic model, and proposed the Leslie-Gower system model. With the development, people gradually discovered that the functional response function should not be a simple linear function, and a more reasonable functional response function should be nonlinear and bounded. C. S. Holling proposed three bounded functional response functions [4, 6, 7, 9, 13, 24]. The results of the above population research include stability, the existence of limit cycles, bifurcation and other issues [5, 11, 14, 20].

The hydra effect is a phenomenon in which population balance or time average density increases when the mortality rate of the population increases [1,2]. In [1], the three key mechanisms underlying the hydra effect were proposed by Abrams. The two latter mechanisms were investigated in [12]. However, this mechanism has been determined in the theoretical research conducted by [16], in which they analyzed predator-prey models with Holling II and III functional responses. Regarding the hydra effect, the paper by Strevens and Bonsall [17] discussed harvesting strategies

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in a host-parasitic wasp complex population system. In [3], a diffusive predator-prey model with functional response function was studied

$$\begin{cases} \frac{\partial R}{\partial t} = D_R \frac{\partial^2 R}{\partial x^2} + rR(1 - \frac{R}{K}) - F(R)C, \\ \frac{\partial C}{\partial t} = D_C \frac{\partial^2 C}{\partial x^2} + ef_{RC}F(R)C - m_C C - q_C C^2, \end{cases} \tag{1.1}$$

where

$$F(R) = \frac{a_{CR}R^n}{1 + a_{CR}Th_{CR}R^n}. \tag{1.2}$$

The authors have shown the occurrence of the hydra effect in some of its steady-state dynamics through numerical simulations. Substituting equation (1.2) into system (1.1), system (1.3) can be obtained. In this paper, we will analyze the dynamic properties of the model (1.3). The system is as follows:

$$\begin{cases} \frac{\partial R}{\partial t} = D_R \frac{\partial^2 R}{\partial x^2} + rR(1 - \frac{R}{K}) - \frac{a_{CR}R^n C}{1+a_{CR}Th_{CR}R^n}, \\ \frac{\partial C}{\partial t} = D_C \frac{\partial^2 C}{\partial x^2} + \frac{ef_{RC}a_{CR}R^n C}{1+a_{CR}Th_{CR}R^n} - m_C C - q_C C^2, \end{cases} \tag{1.3}$$

and the Neumann boundary condition

$$\begin{aligned} R_x(x,t) = C_x(x,t) = 0, R_x(l,t) = C_x(l,t) = 0, t > 0, \\ R(x,0) = R_0(x) \geq 0, C(x,0) = C_0(x) \geq 0, x \in [0,l], \end{aligned} \tag{1.4}$$

where R and C are the density of the prey and the density of the predator respectively, and r is the inherent growth rate of the prey R. K is the environmental capacity of the prey R. m_C is the per capita mortality of the species C that is not related to density, and q_C is per capita mortality of species C related to density. D_R and D_C are the diffusion coefficients of species R and C respectively, and ef_{RC} is the conversion coefficient from species R to species C. a_{CR} is the attack coefficient of species C against species R, and Th_{CR} is the effect time of species C on species R. All parameters are strictly positive constants.

In this paper, we analyze the Turing instability and the existence of Hopf bifurcations of the system (1.3), when $n = 1$. The existence of the hydra effect is shown, when the system (1.3) is undergoing the state bifurcation. The structure of this article is arranged as follows: In Section 2, by analyzing the characteristic equation of the coexistence balance system, we clarity conditions for the existence of Turing unstable and Hopf bifurcations of a diffusive predator-prey system. In addition, we also determine the critical Turing bifurcation and Turing instability curves in the parameter plane. Then, numerical simulations are explained to support the existence of the hydra effect and other theoretical analysis results in Section 3. Finally, in Section 4, we discuss and conclude.

2. Turing instability and Hopf bifurcation analysis

2.1. Existence of positive equilibrium

In this section, we discuss the existence of the positive equilibrium point of system (1.1). In the system (1.1), we set

$$\begin{cases} f(R,C) = rR(1 - \frac{R}{K}) - F(R)C = 0, \\ g(R,C) = ef_{RC}F(R)C - m_C C - q_C C^2 = 0. \end{cases} \tag{2.1}$$