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Noise-induced Transitions for an SIV Epidemic Model with Medical-resource Constraints*

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Abstract We consider a stochastically forced epidemic model with medical-resource constraints. In the deterministic case, the model can exhibit two type bistability phenomena, i.e., bistability between an endemic equilibrium or an interior limit cycle and the disease-free equilibrium, which means that whether the disease can persist in the population is sensitive to the initial values of the model. In the stochastic case, the phenomena of noise-induced state transitions between two stochastic attractors occur. Namely, under the random disturbances, the stochastic trajectory near the endemic equilibrium or the interior limit cycle will approach to the disease-free equilibrium. Besides, based on the stochastic sensitivity function method, we analyze the dispersion of random states in stochastic attractors and construct the confidence domains (confidence ellipse or confidence band) to estimate the threshold value of the intensity for noise caused transition from the endemic to disease eradication.

Keywords Epidemic model, Medical-resource constraints, Noise-induced transitions, Stochastic sensitivity, Confidence domain.

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1. Introduction

Mathematical models play an indispensable role in epidemiological research. For instance, Sun et al., [16] analyzed an SIS model incorporating the effects of awareness spreading on epidemic. In the past few decades, scholars have proposed various epidemic models to explore the complex spreading process and test the effectiveness of disease control after the introduction of interventions [7–9,20,26]. In the classical epidemic models, it is usually assumed that the medical resources such as drugs, vaccines and hospital beds are very sufficient for the infectious disease. However, the reality is that medical resources are usually limited. In order to investigate the dynamics of disease transmission under the absence of medical resources, some epidemic models with medical-resource constraints have been proposed. For example, Zhang et al., [25] introduced a saturated treatment function in an SIR model to describe the limited medical resources. Zhu et al., [14] defined a more complex saturated function to characterize the impact of hospital beds on disease control, and their results show that the model can experience a sequence of bifurcations

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including Hopf bifurcation, saddle-node bifurcation, backward bifurcation. Wang et al., [19] first adopt a piecewise-defined treatment function to simulate a limited capacity for treatment. Then, they further modified the treatment function into the following form [18]

$$T(I)=\begin{cases} rI, & 0\le I\le I_0,\\ rI_0, & I>I_0,\end{cases}$$

which means that the treatment rate is proportional to the case number before the capacity of treatment is reached, then take its maximum value rI_0. Recently, using a similar idea as described in Ref. [18], Wang and Xiao et al., [17] has put forward an SIV epidemic model with a non-smooth but continuous function that depicts vaccination strategies. Their analysis suggested that this model undergoes the following three types of bistability: between two internal equilibrium, between one disease-free equilibrium and one endemic equilibrium, or between one disease-free equilibrium and one limit cycle.

Although the above deterministic models based on constant environmental settings can always capture some hallmarks of disease transmission, unexpected stochastic factors such as resource availability, humidity and temperature, play an indispensable role in the spread of disease. For instance, Mecenas et al., [11] found that the replicability and the transmission capacity of the COVID-19 virus goes hand in hand with different weather conditions such as temperature and relative humidity. In addition, the unpredictability of human-to-human contact is also one of the stochastic drivers in the spread of epidemics when populations are small [15]. Therefore, it is necessary to probe the influence of noisy environmental fluctuations on the disease transmission. To this end, Lan et al., [10] put forward a SIRS epidemic model disturbed by a noisy environment and explored the influence of hospital resources on the final scale of infection. Cai et al., [5] investigated a SIRS model subject to stochastic noise and revealed that environmental fluctuations can restrain disease outbreaks.

For highly nonlinear models, the stochasticity of the environment may push the system back and forth between two metastable states. In recent years, the work related to noise-induced transitions has been extensively considered by mathematicians, ecologists and physicists in various fields. For example, in the field of meteorology, Yang et al., [23] used the maximum possible trajectory to simulate the sudden jump behavior of a stochastic thermohaline circulation model between initial and final states. In the field of biology, Chen et al., [6] multiplicatively introduced non-Gaussian noise into a genetic regulatory model and implemented high-order perturbation expansion technique to calculate the average transition time of protein concentration from low to high. In the field of ecosystem, Scheffer et al., [13] proposed various warning signals for the bifurcation-induced critical transitions to avoid the undesirable state of ecosystem. Xu [21] probed the dynamics mechanisms behind the population size oscillatory transition in a predator-prey model.

For the noise-induced state transition phenomenon, a question worth exploring is how large is the critical noise intensity that can cause a system state transition? The stochastic sensitivity function (SSF) method proposed by Bashkirtseva et al., [1, 12] can effectively answer this problem. This method employs confidence ellipse or confidence band to visualize the spatial arrangement of random states near the deterministic attractors such as the endemic equilibrium or the interior limit cycle. Therefore, we can regard the noise intensity when the confidence ellipse