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Spatial Dynamics of a Diffusive Prey-predator Model with Stage Structure and Fear Effect*

Nana Zhu[1] and Sanling Yuan[1,†]

Abstract To understand the influence of fear effect on population dynamics, especially for the populations with obvious stage structure characteristics, we propose and investigate a diffusive prey-predator model with stage structure in predators. First, we discuss the existence and stability of equilibrium of the model in the absence of diffusion. Then, we obtain the critical conditions for Hopf and Turing bifurcations. Some numerical simulations are also carried out to verify our theoretical results, which indicate that the fear can induce the prey population to show five pattern structures: cold-spot pattern, mixed pattern with cold spots and stripes, stripes pattern, hot-spot pattern, mixed pattern with hot spots and stripes. These findings imply that the fear effect induced by the mature predators plays an important role in the spatial distribution of species.

Keywords Prey-predator model, Fear effect, Stage-structured model, Hopf bifurcation, Turing bifurcation, Pattern formation.

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1. Introduction

For a long time, studying the dynamic behaviors of prey-predator models has been a major topic in both ecology and evolutionary biology [23,26]. It has been long believed that the predators can impact prey populations only through directly killing. However, many theoretical biologists [3,8,9] have argued that indirect effects caused by the fear effect induced by predation may play a more important role on the prey population.

Almost all animals respond risk caused by predators and show various anti-predation behaviours such as adjusting foraging behaviors, changing their habitat usage, guard and physiological changes [4, 13, 14, 17]. For example, the prey may choose to give up the primal high-risk habitat and migrate to the low-risk habitat, when they feel risk caused by predators [4], which can cause a large loss if the quality of the low-risk habitats is worse than that of the primal one. In addition, individuals at different stages may be exposed to different levels of risk and therefore react differently. For example, breeding birds will fly away from nests,

†the corresponding author.
Email address: Sanling@usst.edu.cn (S. Yuan), Nana_zhu95@163.com (N. Zhu)
[1]College of Science, University of Shanghai for Science and Technology, Shanghai 200093, China
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leaving immature birds in danger and taking less care of them, as the mature birds feel dangerous. Even the transient absence of mature birds may lower survival probability of immatures, because immatures may undergo less suitable living environment and face more higher risk of predators [4]. In that case, although the odds of survival for mature birds have increased in short-term, the whole fitness of birds species will decrease, because the fear by predation may cause a reduction in their reproduction [20]. Zanette [24] also conducted some field experiments on song sparrows throughout the breeding season, using electric fences to test direct predation on both young and adult song sparrows. There is no directly killing in all experiments, but the vocal cues of the predators broadcast were used to imitate predation risk in the wild. They tested two groups of female song sparrows, among which one group was exposed to the predator sounds, and the other one was not. The researchers [24] found the one exposed to the predators voice reduced 40 percent fewer offspring than the other group, because fewer eggs were laid and fewer nestlings survived. Therefore, the anti-predator behaviour of prey may be helpful in increasing the probability of survival in a short term, but can cause large costs on reproduction in the long term [4].

On the other hand, the predators may also exhibit different predation abilities at different stage of their growth. For example, there exists a type lion living in South America. Only the mature lions can attack and capture buffaloes as their food, while the immature lions get living resources depending on their parents because they have no ability to attack the buffaloes until they become almost one year old [15]. Notice that in this situation, the fear perceived by prey is only from the mature predators. Therefore, it is essential to divide the predator individuals into different stages when modeling the interaction dynamics between prey and predators. As far as we know, the models with stage structure in prey-predator population have been widely investigated in the following pieces of literature (see e.g., [1, 5, 10, 25]). However, there are relatively few investigations for the stage structure predator-prey models with fear effect, especially for the diffusive predator-prey models with both fear effect and stage structure in predators. To this end, in this paper, we will focus on exploring the influence of fear induced by mature predators on the spatial distribution of prey species by discussing a diffusive prey-predator model with stage structure in predators.

Let $u(x,t)$, $v_1(x,t)$, and $v_2(x,t)$ be respectively the densities of the prey, the juvenile and mature predators at position x and time t. We suppose the prey population grows logistically in the absence of predators, i.e., the per capital rate of prey is $r\big(1-\frac{u}{m}\big)$, where r is the intrinsic growth rate of the prey, m is the environmental carrying capacity for the prey population. The mature predators catch the prey by complying with Holling-type II functional response $\frac{auv_2}{b+u}$ [6], where a stands for the maximal prey consumption rate of a mature predator individual, b represents the half-saturation constant. Since only the mature predators can attack the prey species, the fear perceived by prey is just from the mature predators. Thus, the fear function can be written as $\frac{1}{1+k_1v_2}$, where k_1 stands for the fear level, and therefore, the prey population grows with the rate of $\frac{ru}{1+k_1v_2}\big(1-\frac{u}{m}\big)$. Assume that c is the per capita birth rate of predators, then $\frac{cauv_2}{b+u}$ describes the birth rate of the juvenile predators. As we know, not all the juvenile predators can mature into adults, say the lions in South America we have mentioned above. We suppose the maturation rate of the juvenile predators as α and their death rate is q_1. Moreover, following [2], we assume that the death rate of mature predators is