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A Study on Oscillatory and Asymptotic Nature of Impulsive Neutral Differential Equations of Order Three*

Abdur Raheem[1,†], Asma Afreen[1] and Areefa Khatoon[1]

Abstract In this paper, we consider a class of third-order neutral impulsive differential equations. An equivalent class of neutral differential equations is obtained by using a suitable substitution. Some new oscillation results are proved. Moreover, we discuss the asymptotic behavior of the solution. The results presented here are illustrated via examples.

Keywords Neutral differential equations, Impulsive conditions, Oscillation criteria, Asymptotic behavior.

MSC(2010) 34K11, 34K40, 34G10, 34G45, 35R12.

1. Introduction

Here, we establish the oscillation results for the following model of neutral impulsive differential equation of order three:

$$\begin{cases} \Big(s_1(t)\upsilon^{(2)}(t)\Big)^{(1)} + s_2(t)u^{(1)}(\eta_2(t)) = 0, \quad t \neq t_p, \\ u^{(r)}(t_p) - u^{(r)}(t_p^-) = d_p u^{(r)}(t_p^-), \quad r = 0, 1, 2, \\ \qquad\qquad\qquad\qquad\qquad\qquad p = 1, 2, 3, \dots, \end{cases} \tag{1.1}$$

where $\upsilon(t) = u(t) + \alpha u(\eta_1(t))$, $\eta_1(t) \leq t$, $\eta_2(t) \leq t$, $t > t_0$, $\alpha > 0$, and $\upsilon^{(r)}(t)$ denotes the derivative of order r with respect to t.

It is well-known that the motions on the earth are not always uniform, as various kinds of resistance appear during the motions. For example, suppose high-intensity forces act for a short duration of time. In that case, motions caused by these forces are called impulsive motions, and the differential equations describing these motions are called impulsive differential equations.

The differential equations with impulsive effect can be used to simulate those discontinuous processes in which impulses occur. Therefore, it has become an important tool to handle the real process of mathematical models and phenomena

†the corresponding author.
Email address: araheem.iitk3239@gmail.com (A. Raheem), areefakhatoon@gmail.com (A. Khatoon), afreen.asma52@gmail.com (A. Afreen)
[1]Department of Mathematics, Aligarh Muslim University, Aligarh–202002, India

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such as optimal control, electric circuit, biotechnology, population dynamics, fractals, neural network, viscoelasticity and chemical technology. For more details on impulsive differential equations, please refer to [15]. One of the main advantages of the impulses can be seen in [23], as they provided the model in which the mass point might oscillate in the presence of impulsive effect and in the absence of impulsive effect, the mass point did not oscillate. For more work on impulsive effect, please refer to [7, 14, 19].

In 1989, some researchers started to investigate the oscillatory nature of differential equations with impulses, and they were at the initial stage of development. Later on, the authors of [7,9–11,18,23] extended the study of oscillation to parabolic and hyperbolic impulsive partial differential equations. A hybrid evolution system with impulsive conditions has been studied by Sadhasivam and Deepa [21]. In the last few decades, many researchers [2, 4, 13, 20, 22, 25, 29] have applied the Riccati technique to the study of the oscillatory behavior for various types of second-order differential equations. Some works on the oscillatory and asymptotic behavior of the solutions to higher-order impulsive differential equations have been carried out in [8, 16, 27]. Li [16] investigated the oscillatory and asymptotic behavior of the solutions to a higher-order delay differential equation with impulses by using comparison results with an associated non-impulsive delay differential equation.

Basic definitions and results on oscillation for neutral type differential equations can be found in [5,15]. Due to the wide applicability of neutral differential equations in various fields of science and engineering, there is a great interest in obtaining new oscillation criteria for a different types of differential equations (see, for instance, [2–4,6,12,13,17,22,25,26,29]). We have often seen that even non-impulsive neutral delay differential equations may have solutions of oscillatory nature due to some additional control. An improved sufficient condition for the oscillation and asymptotic stability has been obtained in this paper [26]. Guan and Shen [12] examined the oscillation criteria of a first-order impulsive differential equation with variable delays. Oscillation theorems for third-order delay differential equations were discussed by Tiryaki and Aktas [24]. Arul and Shobha [2] generalized neutral differential equations of order two and presented some new oscillation criteria by using Riccati transformation under some conditions.

Recently, Zhang and Li [28] have studied the oscillatory behaviour of the solutions to second-order impulsive neutral dynamic system with positive and negative coefficients. Moreover, in [22], the authors have presented some new necessary and sufficient conditions for the oscillation of a class of second-order neutral delay impulsive differential equations.

Motivated by all the above works, we obtain some new oscillation results of impulsive neutral differential equations of order three by converting them to the non-impulsive neutral differential equations. Also, there are only a few papers that deal with this technique.

The rest of this paper is organized as follows. Section 2 contains some basic lemmas and assumptions, which are required for the next sections. In Section 3, some new-type oscillation results are obtained for problem (1.1) by using Riccati transformation. In the last section, the results are illustrated by examples.

2. Preliminaries and assumptions

Throughout the paper, we consider the following assumptions: