Commun. Math. Anal. Appl.
doi: 10.4208/cmaa.2022-0018

Vol. **1**, No. 4, pp. 568-589
November 2022

Boundedness and Asymptotic Stability in a Chemotaxis Model with Signal-Dependent Motility and Nonlinear Signal Secretion

Xinyu Tu[1,*], Chunlai Mu[2], Shuyan Qiu[3] and Jing Zhang[2]

[1] *School of Mathematics and Statistics, Southwest University, Chongqing 400715, P.R. China.*
[2] *College of Mathematics and Statistics, Chongqing University, Chongqing 401331, P.R. China.*
[3] *School of Sciences, Southwest Petroleum University, Chengdu 610500, P.R. China.*

Received 3 August 2022; Accepted 26 September 2022

Abstract. In the present study, we consider the following parabolic-elliptic chemotaxis system:

$$\begin{cases} u_t = \nabla\cdot\big(\gamma(v)\nabla u - u\chi(v)\nabla v\big) + \lambda u - \mu u^\sigma, & x\in\Omega, \quad t>0,\\ 0=\Delta v - v + u^\kappa, & x\in\Omega, \quad t>0, \end{cases}$$

where $\Omega\subset\mathbb{R}^n$ $(n\geq 2)$ is a smooth and bounded domain, $\lambda>0,\mu>0,\sigma>1,\kappa>0$. Under appropriate assumptions on $\gamma(v)$ and $\chi(v)$, we obtain the global boundedness of solutions when $\kappa n<2$ or $\kappa n\geq 2, \sigma\geq\kappa+1$, which generalize the previous result to the case with nonlinear signal secretion and superlinear logistic term when $n\geq 2$. Moreover, if adding additional conditions $\sigma\geq 2\kappa$ and μ is sufficiently large, it is shown that the global solution (u,v) converges to

$$\left(\left(\frac{\lambda}{\mu}\right)^{\frac{1}{\sigma-1}},\left(\frac{\lambda}{\mu}\right)^{\frac{\kappa}{\sigma-1}}\right)$$

exponentially as $t\to\infty$.

*Corresponding author. *Email address:* xinyututututu@163.com (X. Tu)

AMS subject classifications: 35K35, 35Q30, 35B40

Key words: Chemotaxis, boundedness, large time behavior, signal-dependent motility, nonlinear signal secretion.

1 Introduction

Chemotaxis, a kind of taxis, which refers to the phenomenon that cells, bacteria or multicellular organisms direct their movements according to certain chemicals. To describe the aggregation phase of amoeba cells in response to the chemical signal emitted by cells, Keller and Segel [20] introduced the following system:

$$\begin{cases} u_t = \nabla \cdot \big(\gamma(v)\nabla u - u\chi(v)\nabla v\big), & x\in\Omega, \quad t>0, \qquad (1.1a)\\ \tau v_t = \Delta v - v + u, & x\in\Omega, \quad t>0, \qquad (1.1b)\end{cases}$$

where $\tau\in\{0,1\}$, $\Omega\subset\mathbb{R}^n$ $(n\geq 1)$ is a bounded domain, $u(x,t), v(x,t)$ denote the cell density and the concentration of chemical signal, $\gamma(v)$, $\chi(v)$ are the cell diffusion function and chemo-sensitivity function respectively, which have the following relation:

$$\chi(v) = (\alpha-1)\gamma'(v), \tag{1.2}$$

where α is the ratio of effective body length to step size.

For the case that $\gamma(v)=d$, $\chi(v)=\chi$, where d and χ are positive constants, (1.1) can be reduced to the minimal Keller-Segel model, whether model (1.1) or its related variants, there has been a large number of research with regard to the existence, boundedness, finite-time blow-up, asymptotic behavior et al. (see the review literatures [2,3,10,12] and the references therein). In particular, considering system (1.1) with nonlinear signal production and general growth source, that is, adding the logistic term $f(u)=\lambda u-\mu u^\sigma$ $(\lambda\in\mathbb{R}, \mu>0, \sigma>1)$ on the right hand side of Eq. (1.1a), and replacing the linear term u in Eq. (1.1b) by u^k $(k>0)$. When $k\geq 1$, Galakhov *et al.* [9] considered the global dynamics of solutions, thereinto, by assuming that $\sigma>k+1$ or $\sigma=k+1, \mu>((nk-2)/nk)\chi$, they obtained the global boundedness result; and this boundedness result was extended to the borderline case $\sigma=k+1$, $\mu=((nk-2)/nk)\chi$, $n\geq 3$ by Hu and Tao [13]; then Xiang [33] removed the restrictions $k\geq 1$ and $n\geq 3$, under the condition $k+1<\max\{\sigma, 1+2/n\}$ or $k+1=\sigma$, $\mu\geq((nk-2)/nk)\chi$, he proved that the solution is globally bounded; afterwards, Xiang *et al.* [30] further extended the result in [33] to the case with nonlinear diffusion function $D(u)$ and nonlinear sensitivity function $S(u)$; lately, considering the chemo-repulsion case, Hu *et al.* [15] established the global bound-