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Is Objectivity Really an Objective Concept?

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Abstract. Objectivity is the name given to covariance with respect to changes in observers by the action of the Euclidean group and is a central concept for mathematical modelling in Physics and Continuum mechanics. However, this notion is delicate and therefore hard to understand by both mechanicians and mathematicians, precisely because it resides at the junction of the two disciplines and hence appeals to knowledge and notions from both fields to become clear. In a first part ("behind objectivity") some classical notions of objectivity and frame-dependence are presented, discussed and critically revisited with the purpose to introduce in a second part ("beyond objectivity") a novel modelling approach in incompatible elasticity, based on intrinsic and geometric concepts, related but distinct from conventional objectivity. This contribution is primarily conceived to assist students, academics and researchers, mathematicians in particular, to find their way through some difficult and often ill-understood concepts of Physics and Continuum mechanics, since objectivity still remains nowadays a profound concept that deserves our attention.

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1 Introduction

Foundational research has the aim of clarifying the roots of our analyses, which roots determine the meaning and significance of our proposals in the description of

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Nature. In this work we would like to introduce and discuss a crucial foundational issue in Continuum mechanics: objectivity. Nature and use of this concept is at the interface between mathematics as a set of theories generating formal languages, and philosophy. Objectivity is inherent to Physics since the ancient Greeks, and with a mathematical formulation since Newton, d'Alembert, Einstein, Poincaré and Lorentz. In Continuum mechanics it was given a central rôle by Noll [31] and innumerous references afterwards (see, e.g., [14, 18, 35, 37–39]).

The concept has been heavily debated during decades until a sort of consensus was reached in the 80ies. The controversy but also the unavoidable polemics arisen from this lack of agreement are possibly due to an excessive promotion of this principle to an axiom, presumably because over the long term Science remains reticent to dogmatic assertions. The consensus about objectivity is that it is a in part a physical principle, and also a modelling tool, that has shown correct and powerful in the modelling process of most of the phenomena observed in Continuum media. Nonetheless, it has been debated that for some media such as polymers, objectivity might hold only in first approximation, as advocated by De Gennes in [1], whereas other authors propose an approach compatible with objectivity, as James or Mariano in [15, 20]. In the same spirit, it was also debated that objectivity might sometimes also mislead or even be unappropriate, unless non-objective tensors are accounted for in the description of some observed phenomena. Therefore, objectivity should rather be accepted as an instrument, rather than an axiom or a postulate, useful to devise behaviour laws, the so-called constitutive relations of the material.

It is important to clarify that objectivity when used to write response functions cannot be considered without being incorporated in the complete system of State equations (in general provided by balance laws) on which Galilean (i.e., inertial) observers agree, and which are Galilean invariant (or Lorentz-Poincaré invariant for relativistic phenomena). Note that Continuum mechanics modelling is to be understood as a theory possibly incorporated in Einstein's theory of Special relativity, without the need to appeal to general covariance principles as required for the description of Space-time in General relativity. Indeed, in the latter, covariance with respect to general change of coordinates is in force, but this fundamental principle is too strong to devise material behaviour laws of Continuum mechanics. Truly, there is a double answer to frame dependence phenomena: first, as a property of matter, such as observed for long and heavy molecules (as for instance for nematic liquid crystalline phases); second, as a property of the frame in which the equations are stated. In the first case, lack of objectivity is inherent to the system, in the second, it is an artefact.

Classical definitions around these concepts are reviewed in a first part, rising some issues or paradoxes that we also try to explain. This first part is written in