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A Mathematical Study for the Stability of Two Predator and One Prey with Infection in First Predator using Fuzzy Impulsive Control

Khushbu Singh[1] and Kolla Kaladhar[1,*]

[1] *Department of Mathematics, National Institute of Technology, Warangal-506004, India*

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Abstract. In this study, we develop a set of ordinary differential equations that represents the dynamics of an ecosystem with two predators and one prey, but only the first predator population is affected by an infectious disease. The Lotka-Volterra predator-prey system's model stability have been examined using the Takagi-Sugeno (T-S) impulsive control model and the Fuzzy impulsive control model. Following the formulation of the model, the global stabilities and the Fuzzy solution are carried out through numerical simulations and graphical representations with appropriate discussion for better understanding the dynamics of our proposed model.

AMS subject classifications: 92D25, 92D30, 92D40, 49N25

Key words: Prey-predator system, T-S model, stability, eco-epidemiology.

1 Introduction

One of the most popular subjects in biomathematics is population dynamics. There has always been an unique interest in the study of population evolution, beginning with populations of a single species and progressing to more realistic models

*Corresponding author.
Emails: kaladhar@nitw.ac.in (K. Kaladhar), khushbu91@student.nitw.ac.in (K. Singh)

where various species coexist and communicate with one another in the same ecosystem. Between these, we can find models that look at predator-prey relationships, symbiosis, or competitive connections. Since the well-known Lotka-Volterra model was developed and the major issues with ecological processes were resolved [1, 36], mathematical models are frequently used by applied mathematicians to analyse the intricate interactions between predators and prey. The classical ecological models of interacting populations typically have focused on two species. The literature has looked into continuous time models of two interacting species in great detail [16]. These models can only display the following two basic patterns mathematically: approach to a limit cycle or a steady state [17]. However, it has been found that ecological groups in nature have extremely complicated dynamical tendencies. According to Price et al. [27] community behaviour needs to be based on at least three trophic levels. There are reports of more intricate patterns in three species continuous time models [2, 14, 20, 21, 26, 32, 33, 38].

One of the most intriguing areas in mathematical biology is the interaction between predators and prey. The well-known Lotka-Volterra predator-prey model is the first mathematical representation of the interaction between predators and prey [37], which is a two-species model. Some scholars have noted that population models with two species can't accurately capture the real world [15, 31], and models with three or more species can only depict a significant number of crucial behaviours. The advancement of mathematics also demonstrated that three-species food chain models have significantly more detailed features than two-species models [8, 34].

Since the impact of infectious diseases on the ecological system regulates population size, researchers have recently become more interested in the fusion of ecology and epidemiology. There are a lot of prey-predator models that have infectious infections. The dynamics of the prey-predator system with disease in the prey and predator populations were hypothesised and examined by [6, 7], Haque and Venturino [22], Haque et al. [23, 24], Xiao and Chen [40, 41], Zhou et al. [42], Tewa [12], Hethcote [9], Hudson [28], recently, Deng [25] etc. Additionally, numerous research studies have explored the dynamic behaviour of the predator-prey system with infection in the predator population. [7]. There are also several scholars who have studied eco-epidemic models where predator populations are infected through consuming prey, such as Anderson and May [30], Hadeler and Freedman [13] etc. The dynamics of a predator-prey model with disease in both prey and predator populations were proposed by Hsieh and Hsiao [43]. Additionally, some researchers have developed eco-epidemic models with optimal control [3] and with temporal delays [11, 18].

We have witnessed rapidly growing interested in fuzzy control in recent years. This is largely sparked by the numerous successful applications fuzzy control has