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Energy Decay for a Type of Plate Equation with Degenerate Energy Damping and Source Term

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Abstract. In this paper, we mainly investigate the initial boundary value problem for a plate equation with non-local degenerate energy damping term and source term. By using potential well and Nakao's inequality, we establish the global existence and the energy decay rate when the initial data is starting in the stable set. Finally, we derive some further estimate on the stability.

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1 Introduction

In this paper, we investigate the initial boundary value problem for the following plate equation with nonlocal energy damping

$$\begin{cases} u_{tt}+\Delta^2 u - M(\|\nabla u\|_2^2)\Delta u - \left(\|\Delta u\|_2^2+\|u_t\|_2^2\right)^q \Delta u_t = |u|^{p-2}u, & (x,t)\in\Omega\times(0,\infty), \\ u=\dfrac{\partial u}{\partial \nu}=0, & (x,t)\in\partial\Omega\times\mathbb{R}^+, \\ u(x,0)=u_0(x),\, u_t(x,0)=u_1(x), & x\in\Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ ($n\geq 1$) with sufficiently smooth boundary, $q\geq 1$, $p>2$, ν is the unit outer normal to $\partial\Omega$. $M(s)$ is a continuous function on $[0,\infty)$, $u_0(x)$ and $u_1(x)$ are given initial data in Section 2.

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For the limit case ($q=0$, non-degeneracy), Eq. (1.1) turns to be the classical extensible beam/plate model with strong damping $-\Delta u_t$, which is widely investigated in the literature. For the related research on this aspect, one can see [1,2] and references therein.

Recently, the evolution equations with nonlocal dissipative effects are extensively investigated by many researchers in different contexts. In fact, the kind of nonlocal dissipative effect $M(\|\nabla u\|_2^2)u_t$ was first introduced by Lange and Perla Menzala [3] for the beam equation,

$$u_{tt}+\Delta^2 u+N(\|\nabla u\|_2^2)u_t=0 \quad \text{in}\,\mathbb{R}^n\times\mathbb{R}^+. \tag{1.2}$$

Cavalcanti *et al.* [4] studied Eq. (1.2) by adding a memory item $-(g*\triangle u)(t)$, where energy decay was obtained by multiplier technique. Jorge Silva and Narciso [5] considered the following model

$$u_{tt}+\Delta^2 u+N(\|\nabla u\|^2_{L^2(\Omega)})u_t+f(u)=h(x)$$

in the bounded domain Ω, and proved the existence of the global attractor with finite dimension. Another kind of nonlocal fractional damping term is given by

$$N(\|\nabla u\|_2^2)(-\triangle)^\theta u_t, \quad 0\le\theta\le 1.$$

For the related research on this topic with $N>0$, one can see [6–9] and references therein. We also refer the readers to the references [10–12] on the nonlocal nonlinear damping term $N(\|\nabla u\|_2^2)g(u_t)$.

Now, we turn back to our model with nonlocal energy damping in relation to the flight structure, whose basic one-dimensional (time) dynamics can be described as

$$x''(t)+\omega^2(x)+\gamma D(x(t),x'(t))=0. \tag{1.3}$$

As discussed in [13], the following damping model (also called energy damping)

$$D(x(t),x'(t))=\left(\frac{\omega^2}{2}[x(t)]^2+\frac{1}{2}[x'(t)]^2\right)^q x'(t), \quad q>0,$$

has been considered. Denoting $E(t)=\frac{\omega^2}{2}[x(t)]^2+\frac{1}{2}[x'(t)]^2$ as the energy functional, the model (1.3) turns into the following

$$x''(t)+\omega^2(x)+\gamma[E(t)]^q x'(t)=0.$$

In this way, one can get a new class of dissipative models with nonlocal energy damping. Some researchers have successively proposed several models corresponding to one-dimensional beam equations [14–17]. For example, we highlight the next prototype:

$$u_{tt}-2\varsigma\sqrt{\lambda}u_{xx}+\lambda u_{xxxx}-\gamma\left[\int_{-L}^{L}(\lambda|u_{xx}|^2+|u_t|^2)\mathrm{d}x\right]^q u_{xxt}=0. \tag{1.4}$$