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Flocking of Multi-particle Swarm with Group Coupling Structure and Measurement Delay*

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Abstract We investigate the flocking conditions of a group coupling system with time delays, in which the communication between particles includes inter-group and intra-group interactions, and the time delay comes from the theory of moving object observation. As an effective model, we introduce a system of nonlinear functional differential equations to describe its dynamic evolution mechanism. By constructing two differential inequalities on velocity and velocity fluctuation from a continuity argument, and using the Lyapunov functional approach, we present some sufficient conditions for the existence of asymptotic flocking solutions to the coupling system, in which an upper bound of the delay allowed by the system is quantitatively given to ensure the emergence of flocking behavior. All results are novel and can be illustrated by using some specific numerical simulations.

Keywords Time-asymptotic flocking, Group coupling, Intra-group and inter-group interaction, Measurement delay.

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1. Introduction

"Flocking" is a collective behavior that widely exists in biological populations and human society such as migration of birds, directional movement of fish, hunting by wolves and gathering of bacteria [15,33,34]. The research of biological flocking dates back to the simulation of bird behavior in 1987 [36]. Based on this, subsequent researchers proposed a series of group motion models such as Vicsek model [40], Couzin model [7], Cucker-Smale model [9, 10] and their variants. Among them, the pioneering progress was that Cucker and Smale had proposed a second-order nonlinear dynamic model (C-S model) with a Newtonian interaction function to analyze the flocking mechanism of a multi-particle swarm in [9, 10]. The flocking dynamics and the related topics based on the C-S model have been extensively studied from many perspectives such as collision avoidance [4, 8, 25, 30], random effects [1, 6, 11, 14, 16, 21, 32, 39], generalized flocking [31], fixed-time flocking [26, 41] and multi-cluster flocking [43]. It is worth noting that in the work mentioned above, the interaction weight function has a unified form, either the symmetric

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type proposed in [9, 10] that depends on the metric distance, or the asymmetric type proposed in [31] that depends on the relative distance scaling.

However, as the scale of system nodes increases, the interaction modes between nodes become complex and diverse. To solve the dynamic mechanism modeling, some scholars have adopted a direct method, using Vlasov-type equations to describe the infinite size system [17–20]. However, from a practical perspective, the number of nodes in a biological population or engineering system cannot be infinite such that a swarm of bees or micro drone systems, resulting in the model proposed in [17–20] are no longer applicable. In addition, we know that real systems are often affected by the things that are not fully understood such as the most common time delays first proposed in [27]. In fact, time delay has also been widely studied in other fields, accumulating rich theoretical results [13, 24, 37]. Therefore, these deterministic models need to be extended for more complex changes. Given this, from the perspective of mathematical mechanism, we use the idea of chunking to highlight the diversity of interaction functions in the system, and further preliminarily assume that the system interaction mode consists of inter-group interaction and intra-group interaction, as shown in Figure 1 in Section 4, and then consider the time delay derived from the theory of moving object observation proposed in [2].

Next, we briefly discuss our continuous time flocking model involving time delay. Let $(x_i, v_i) \in \mathbb{R}^{2d}$ be the position and velocity coordinates of the i-th particle, and be governed by

$$\begin{cases} \dot{x}_i(t) = v_i(t), \ i \in \mathcal{N}, \\ \dot{v}_i(t) = \alpha \sum\limits_{j\in\mathcal{N}_1} \psi(r_{ji})\widetilde{v}_{ji}(t) + \kappa \sum\limits_{j\in\mathcal{N}_2} a_{ij}\widetilde{v}_{ji}(t), i \in \mathcal{N}_1, \\ \dot{v}_i(t) = \kappa \sum\limits_{j\in\mathcal{N}_1} a_{ij}\widetilde{v}_{ji}(t) + \beta \sum\limits_{j\in\mathcal{N}_2} \psi(r_{ji})\widetilde{v}_{ji}(t), i \in \mathcal{N}_2, \end{cases} \tag{1.1}$$

where $\mathcal{N} := \{1, 2, \cdots, N\}$, $\mathcal{N}_1 \cap \mathcal{N}_2 = \emptyset$ and $\mathcal{N}_1 \cup \mathcal{N}_2 = \mathcal{N}$. In addition, $N_i = |\mathcal{N}_i|(i = 1, 2)$, $N = |\mathcal{N}|$ and $N_1 + N_2 = N$; $r_{ji} = \|x_j - x_i\|$, $\widetilde{v}_{ji}(t) = v_j(t-\tau) - v_i(t)$ for $j \in \mathcal{N}$, and $\tau > 0$ is the time delay derived from the theory of moving object observation. In (1.1), $\alpha > 0$ and $\beta > 0$ are intra-group coupling strengths; $\kappa > 0$ is inter-group coupling strength; ψ the intra-group interaction function, which is bounded, positive, non-increasing and Lipschitz continuous on $\mathbb{R}^+$ with $\psi(0) = 1$; a_{ij} represents the interaction strength between groups, and is a bounded positive constant, assuming that $0 < a_{ij} = a_{ji} \le 1$ for $i \in \mathcal{N}_k, \ j \in \mathcal{N}_l$ with $k \ne l \in \{1, 2\}$ is satisfied. Notice that the well-posedness of the time-delayed system (1.1) can be found in [22], which proves the existence and uniqueness of the classical solutions to system (1.1) for the given continuous initial conditions.

Specifically, system (1.1) can be degenerated into the C-S system proposed in [9, 10], and the threshold phenomena between global and local flocking has been observed. With our observations, if $\kappa = 0$ and $\tau = 0$ in (1.1), then there is no interaction between the two groups in the system, which can be regarded as two independent two groups that do not interfere with each other. In this case, some sharp conditions for flocking behavior can be obtained according to the classic flocking results in [9, 10, 19], as illustrated in Appendix A. Since then, these flocking estimates have been generalized for the generally non-increasing communication weight in [19], and the model considered is (1.1) with $\mathcal{N}_k = \emptyset(k = 1$ or $2)$ and $\tau = 0$. For another, notice that typical time delays include transmission delay [3, 5], processing delay [42] and their combination [27, 35], and the C-S model with time