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Traveling Epizootic Waves of a Fox Rabies Model with Small Spatial Diffusion*

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Abstract In this paper, to describe the spread of fox rabies, a degenerate SEI epidemic model with small spatial diffusion equipped by infectious foxes due to rabies is investigated. In particular, the existence of traveling waves is established by the geometric singular perturbation theory for the larger speeds, while the non-existence of traveling wave is still derived for the smaller speeds. Moreover, some numerical simulations are implemented to illustrate the propagation dynamics driven by traveling waves.

Keywords Fox rabies, traveling waves, geometric singular perturbation theory, small spatial diffusion

MSC(2010) 35K57, 35R35, 92D25, 92D40.

1. Introduction

This paper investigates the propagation dynamics in the following reaction-diffusion equation modeling the rabies among foxes:

$$\begin{cases} \dfrac{\partial S}{\partial \tau} = (a-b)S\left(1-\dfrac{N}{K}\right) - \beta IS, \\ \dfrac{\partial E}{\partial \tau} = \beta IS - \sigma E - \left[b + (a-b)\dfrac{N}{K}\right]E, \\ \dfrac{\partial I}{\partial \tau} = D_0 \dfrac{\partial^2 I}{\partial x^2} + \sigma E - \alpha I - \left[b + (a-b)\dfrac{N}{K}\right]I, \end{cases} \tag{1.1}$$

where

$$N = S + E + I$$

is the density of total fox population, while $S(x,\tau), E(x,\tau)$ and $I(x,\tau)$ are the densities of susceptible foxes, infected but non-infectious, i.e., exposed rabid foxes, and infectious foxes at location x and time τ respectively; σ is the average rate at which infected foxes become infectious; β is the transmission coefficient of fox

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rabies; α is the mortality of infectious foxes; a, b are the birth rate and the intrinsic death rate; K is the environmental carrying capacity. Thus, the term $(a-b)N/K$ describes the depletion of the food supply by all foxes. In particular, the diffusion coefficient D_0 is introduced on behalf of the additional spatial activity of infectious foxes due to rabies, which both the susceptible and exposed foxes do not have. Note that all parameters in the model are positive constants.

As is well-known, rabies is a serious disease threatening human health since it could be easily transmitted from infectious foxes to domestic animals, and then to humans. For example, despite of various efforts to stop it by hunting foxes and vaccinating them, the wave of epizootic in Europe was widely dispersed in northern France in 1980 [15]. Indeed, evidences suggest that epidemics such as fox rabies always spread spatially in a way like traveling waves [17]. Thus, there are natural reasons to comprehend the transmission behavior of rabies, in particular, the propagation dynamics of epidemics [12].

In fact, Anderson et al. [1] proposed earlier a kinetic model of fox rabies without the spatial diffusion to describe the dynamics of the spread of rabies. Furthermore, Murray et al. [17] adopted an asymptotic analysis method to study the first-order approximation traveling waves. Different from such studies, this paper accurately analyzes the existence of traveling waves by selecting the small diffusion coefficient D_0 as the perturbation parameter and combining with the geometric singular perturbation method, as employed by Szmolyan [20] and Gourley [8]. The readers are referred to Ruan and Xiao [19], Wang and Wang [21] and Pang and Xiao [18] for more existence or non-existence results of traveling waves established by such a way. As a result, we can therefore draw the conclusion of the dynamic propagation of the system in the space with more practical significance and more abundant phenomena.

Henceforth, in order to simplify the model, we rescale the previous system (1.1) to the following model with non-dimensional quantities by setting $s = S/K, e = E/K, i = I/K, n = N/K, D = D_0/\beta K, t = \beta K\tau, \epsilon = (a-b)/\beta K, \delta = b/\beta K, \mu = \sigma/\beta K$ and $d = (\alpha + b)/\beta K$:

$$\begin{cases} \dfrac{\partial s}{\partial t} = \epsilon(1-n)s - is, \\ \dfrac{\partial e}{\partial t} = is - (\mu + \delta + \epsilon n)e, \\ \dfrac{\partial i}{\partial t} = D\dfrac{\partial^2 i}{\partial x^2} + \mu e - (d + \epsilon n)i, \end{cases} \tag{1.2}$$

where $n = s + e + i$, and ϵ, μ, δ, d and D are positive.

The rest of this paper is structured as follows: First, Section 2 is devoted to prove the non-existence of traveling waves for $R_0 > 1$ and $0 < c < c^*$, while obtaining the existence of traveling waves by the geometric singular perturbation method for $R_0 > 1$ and $c \geq c^*$. Finally, in Section 3, some numerical simulations are implemented to illustrate our main results about the existence of traveling wave solutions of system (1.2).