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Existence and Uniqueness of the Solution for Hilfer Neural Networks with Delays*

Lulu Wang[1], Yu Tian[2,†]

Abstract This paper mainly concerns with a class of nonlinear Hilfer fractional neutral recurrent neural networks with time varying delays. The existence and uniqueness of solutions in the space of weighted continuous functions are established by Banach's contraction principle. Finally, an example is provided to illustrate the application of the obtained results.

Keywords Hilfer fractional differential equation, existence and uniqueness, weighted space of continuous functions, fixed point theory

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1. Introduction

In the past three decades, fractional calculus has received an increasing attention due to its various applications in engineering, mechanics, signal processing, material sciences, etc [1–4]. The related theories about fractional differential equations have been extensively studied by many researchers [5–8]. Additionally, Hilfer fractional derivative, which includes Riemann-Liouville fractional derivative and Caputo fractional derivative, was introduced in [1]. After that, a large number of fractional differential equations with Hilfer fractional derivatives were studied [9–13].

On the other hand, neutral neural networks with time delay have aroused naturally in a wide variety of fields like physics, chemistry, control, viscoelastic mechanics, porous media, electromagnetic and polymer rheology, etc. Therefore the issue concerning the existence, uniqueness of solutions of neutral neural networks has been widely discussed by many authors [14–19].

The successive approximation method [20, 21] and the fixed point theorems [20–30] have long been viewed as the main classical methods of studying existence and uniqueness problems in many areas of differential equations. Additionally, compared with fractional differential equations with delays, many dynamical systems not only depend on present and past states but also involve derivatives with delays, and neutral fractional differential equations with delays are often used to describe such systems [23, 31, 32]. Few authors studied the existence and uniqueness for a problem involving Hilfer fractional derivative. For example, C. Kou et

[†]the corresponding author.
Email address: tianyu2992@bupt.edu.cn (Yu Tian), wanglulu2432@163.com (Lulu Wang)
[1,2]School of science, Beijing University of Posts and Telecommunications, Beijing 100876, China

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al. studied the existence and uniqueness on fractional differential equation with Riemann-Liouville fractional derivative [33]. D.F. Luo et.al proved the uniqueness of the solution for the stochastic fractional delay system with Caputo fractional derivative, see [31]. K.M. Furtati et al. studied the existence and uniqueness of global solutions for a class of nonlinear fractional differential equations involving Hilfer fractional derivative, see [34]. However, to the best of our knowledge, the existence and uniqueness of nonlinear Hilfer fractional neutral recurrent neural networks have not been yet developed. In this paper, we will study the existence and uniqueness of a class of nonlinear Hilfer fractional neutral recurrent neural networks with time varying delays by using Banach's fixed point theory.

In this paper, we consider a general class of neural networks with discrete and distributed varying delays which is described by

$$\begin{aligned} D_{0^+}^{\alpha,\beta}\Big[x_i(t)-\sum_{j=1}^{n}q_{ij}x_j(t-\tau(t))\Big] &= \sum_{j=1}^{n}a_{ij}f_j(x_j(t))+\sum_{j=1}^{n}b_{ij}g_j(x_j(t-\tau(t))) \\ &\quad+\sum_{j=1}^{n}l_{ij}\int_{t-r(t)}^{t}h_j(x_j(s))ds, t\in[0,+\infty) \end{aligned} \tag{1.1}$$

or

$$D_{0^+}^{\alpha,\beta}\Big[x(t)-Qx(t-\tau(t))\Big]=Af(x(t))+Bg(x(t-\tau(t)))+W\int_{t-r(t)}^{t}h(x(s))ds,$$

where $D_{0^+}^{\alpha,\beta}$ is the Hilfer fractional derivative with $0<\alpha<1$ and $0\le\beta\le1$, for $i=1,2,3,...,n, x(t)=(x_1(t),x_2(t),...,x_n(t))^T\in\mathbb{R}^n,(x_i(t)\in C([\vartheta,\infty),R))$, are the status vector relating to the neurons; $A=(a_{ij})_{n\times n}, B=(b_{ij})_{n\times n}$ and $W=(l_{ij})_{n\times n}$ represent the connection weight matrix, delayed connection weight matrix and distributed delayed connection weight matrix, respectively; f_j,g_j,h_j are activation functions, $f(x(t))=(f_1(x_1(t)),f_2(x_2(t)),...,f_n(x_n(t)))^T\in\mathbb{R}^n$, $g(x(t))=(g_1(x_1(t)),g_2(x_2(t)),...,g_n(x_n(t)))^T\in\mathbb{R}^n$, $h(x(t))=(h_1(x_1(t)),h_2(x_2(t)),...,$ $h_n(x_n(t)))^T\in\mathbb{R}^n$, and the mappings $f_j(\cdot),g_j(\cdot)$, and $h_j(\cdot)$ are globally Lipschitz continuous with constants α_j,β_j and $\gamma_j>0$, which satisfy $f(0)\equiv0,g(0)\equiv0,h(0)\equiv0$, for $j=1,2,...,n$. Here $\tau(t)$ and $r(t)$ are nonnegative continuous functions that express discrete time varying delay and distributed time varying delay, respectively. Besides, the delays satisfy $\lim_{t\to\infty}t-\tau(t)\to\infty$ and $\lim_{t\to\infty}t-r(t)\to\infty$. The initial condition for the system (1) is given by

$$I_{0^+}^{(1-\alpha)(1-\beta)}\Big[x_i(0)-\sum_{j=1}^{n}q_{ij}x_j(-\tau(0))\Big]=\phi_i(0)-\sum_{j=1}^{n}q_{ij}\phi_j(-\tau(0)), \tag{1.2}$$

$$x(t)=\phi(t),t\in[\vartheta,0], \tag{1.3}$$

where $I_{0^+}^{(1-\alpha)(1-\beta)}$ is the Riemann-Liouville fractional integral operator. Denote $\vartheta=\inf_{t\ge0}\{t-\tau(t),t-r(t)\},t\mapsto\phi(t)=(\phi_1(t),\phi_2(t),,...,\phi_n(t))^T\in C\big([\vartheta,0],L^p_{\mathcal{F}_0}(\Omega;\mathbb{R}^n)\big)$.

The purpose of this paper is to investigate the existence and uniqueness of the solution to the neutral delayed neural networks (1.1) with initial conditions (1.2)-(1.3) through fixed point method. The paper is organized as follows. Some necessary concepts and related lemmas are reviewed in Section 2. In Section 3, we prove the existence and uniqueness of the solution. Examples to illustrate our main results are given in Section 4.