Dynamical Behavior of a Stochastic Nutrient-plankton Food Chain Model with Lévy Jumps*

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Abstract In aquatic ecosystem, plankton populations are easily affected by environmental fluctuations due to the unpredictability of many physical factors. To better understand how environmental fluctuations influence plankton populations, in this paper, we propose and investigate a stochastic nutrient-plankton food chain model with Lévy jumps. Firstly, by constructing a suitable Lyapunov function, we prove that the stochastic model has a unique global positive solution for any given positive initial value. Then, we establish sufficient conditions for the persistence and extinction of plankton. Finally, we provide some numerical simulations to illustrate the analytical results.

Keywords Stochastic nutrient-plankton model, persistence and extinction, Lévy jumps

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1. Introduction

Plankton is the basis of the marine food webs and supports various marine and terrestrial organisms, so it plays a key role in marine atmospheric dynamics [1]. In order to deeply understand the interaction between plankton and nutrient, many mathematical models have been developed in the past decades [2–6]. The nutrient-phytoplankton-zooplankton model incorporates one of the simplest sets of dynamics that usefully describe oceanic plankton dynamics [7]. For example, the author of [8] proposed the following nutrient-plankton food chain model:

$$\begin{cases} \dfrac{\mathrm{d}N(t)}{\mathrm{d}t} = D(N^0 - N) - aP(t)N(t) + \mu_3 P(t) + \mu_4 Z(t), \\ \dfrac{\mathrm{d}P(t)}{\mathrm{d}t} = aP(t)N(t) - bP(t)Z(t) - (\mu_1 + D_1)P(t), \\ \dfrac{\mathrm{d}Z(t)}{\mathrm{d}t} = bP(t)Z(t) - (\mu_2 + D_2)Z(t), \end{cases} \tag{1.1}$$

where $N(t)$, $P(t)$ and $Z(t)$ denote the concentrations of nutrient, phytoplankton and zooplankton at time t, respectively. The parameter a is the maximal nutrient uptake rate of phytoplankton, b is the maximal nutrient uptake rate of zooplankton,

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N^0 is the input concentration of nutrient, D is the washout rate of nutrient, D_1 is the washout rate of phytoplankton, D_2 is the washout rate of zooplankton, μ_1 is the death rate of phytoplankton, μ_2 is the death rate of zooplankton, μ_3 is the nutrient recycling rate from dead phytoplankton, and μ_4 is the nutrient recycling rate from dead zooplankton. From a biological perspective, we assume that $\mu_3 \leq \mu_1$ and $\mu_4 \leq \mu_2$. All parameters are positive.

It is well recognized that randomness can be seen everywhere in nature [9]. Of course, ecological populations are no exception. As revealed by May [10], biological parameters such as the birth rate, death rate and competition coefficient will fluctuate more or less randomly due to the randomness of the environment. For example, Zhao et al. [11] studied a stochastic phytoplankton allelopathy model under regime switching, revealing that the allelopathy effect plays a vital role in the existence of ergodic stationary distribution. Sarkar and Chattopadhayay [12] introduced environmental fluctuations into a phytoplankton-zooplankton system and concluded that the control of the intensity of noise is a key factor for the termination of planktonic blooms. Majumder et al. [13] presented a minimal model of phytoplankton-zooplankton interaction and compared its dynamics with the stochastic version extended by two different stochastic perturbation approaches, revealing that the persistence of species largely depends on the environmental noise. Imhof and Walcher [14] proposed a stochastic chemostat model for single substrate, showing that the white noise may make the microorganism extinct. Based on model (1.1), Yu et al. [15] proposed the following stochastic nutrient-plankton food chain model:

$$\begin{cases} \mathrm{d}N(t) = [D(N^0 - N) - aP(t)N(t) + \mu_3 P(t) + \mu_4 Z(t)]\mathrm{d}t \\ \qquad\quad + \sigma_1 N(t)\mathrm{d}B_1(t), \\ \mathrm{d}P(t) = [aP(t)N(t) - bP(t)Z(t) - (\mu_1 + D_1)P(t)]\mathrm{d}t \\ \qquad\quad + \sigma_2 P(t)\mathrm{d}B_2(t), \\ \mathrm{d}Z(t) = [bP(t)Z(t) - (\mu_2 + D_2)Z(t)]\mathrm{d}t + \sigma_3 z(t)\mathrm{d}B_3(t), \end{cases} \qquad (1.2)$$

where $B_i(t)(i = 1, 2, 3)$ represents the Brownian motion defined in the complete probability space $(\Omega, \mathbb{F}, \mathbb{P})$, and $\sigma_i(i = 1, 2, 3)$ stands for the intensity of white noise.

Meanwhile, the food chain may suffer sudden and serious environmental shocks, such as floods and toxic pollutants, which cannot be accurately described by model (1.2). To improve the situation, scholars turn to use the non-Gaussian Lévy noise to simulate these discontinuous abrupt environmental shocks in nature (see e.g. [16–20]). For example, Zeng et al. [16] considered a stochastic three species food chain model with general Lévy jumps. The criterion on the global stability in the mean with probability one for each species is established. The results showed that the dynamics of the model can be significantly changed by Lévy jumps. Gao et al. [17] investigated a Lotka-Volterra food chain chemostat model in random environment. They proved the existence and uniqueness of the global positive solution and further obtained the conditions for the extinction and persistence in the mean of microorganisms. Lu and Ding [18] considered a stochastic competitive model with infinite delay and general Lévy jumps. Sufficient conditions for stability in time average are established as well as permanence in time average and extinction. Zhu et al. [21] investigated a stochastic nonautonomous Gompertz model with Lévy jumps. The existences of a global positive solution and an explicit solution have