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Mathematical Analysis of SIR Epidemic Model with Piecewise Infection Rate and Control Strategies

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Abstract The limitation of contact between susceptible and infected individuals plays an important role in decreasing the transmission of infectious diseases. Prevention and control strategies contribute to minimizing the transmission rate. In this paper, we propose SIR epidemic model with delayed control strategies, in which delay describes the response and effect time. We study the dynamic properties of the epidemic model from three aspects: steady states, stability and bifurcation. By eliminating the existence of limit cycles, we establish the global stability of the endemic equilibrium, when the delay is ignored. Further, we find that the delayed effect on the infection rate does not affect the stability of the disease-free equilibrium, but it can destabilize the endemic equilibrium and bring Hopf bifurcation. Theoretical results show that the prevention and control strategies can effectively reduce the final number of infected individuals in the population. Numerical results corroborate the theoretical ones.

Keywords Epidemic model, prevention and control strategy, piecewise infection rate, Hopf bifurcation

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1. Introduction

Mathematical model is an important tool of describing the transmission dynamics of infectious diseases and evaluating the control strategies for curbing epidemics, which has become a modern and interesting direction in the infectious diseases field. Currently, the transmission of infectious diseases modeling has been widely studied, including classical SI, SIR compartment models and so on [2, 13, 19, 21]. In the process of infectious diseases modeling, disease incidence plays a key role in the long-term dynamic behaviour of the model. Most traditional models of infectious diseases assume that the incidence is standard or bilinear. However, the saturation incidence sometimes gives a better representation of the dynamics of infectious diseases [7, 15, 18].

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Various dynamic models have been proposed by mathematicians to investigate the spread and evolution of infectious diseases. Chang, Meng and Zhang [1] investigated a nonlinear stochastic SIS epidemic system with multiplicative noise. Qi et al., [20] proposed two new stochastic non-autonomous SEIS epidemic dynamical models with latent and active patients. He, Wang and Huang [8] studied a nonlocal and time-delayed reaction-diffusion epidemic model with the vaccination strategy in a heterogeneous habitat.

As a means of prevention and control strategies, media reports attract people's attention by reporting the number of infected individuals, deaths and preventive policies. Then, people can reduce the risk of infection by wearing masks and washing their hands frequently. Many researchers have proposed some different mathematical functions or models to describe the impact of media reports on infection rate [3, 22, 27]. Moreover, threshold policy is often used in the control measures [3, 24]. It is often the case that when the number of cases exceeds a certain value, the government will remind the public of avoiding contacting infectious sources and suspected patients for the purpose of crisis awareness of the disease. Thus, it will effectively reduce the incidence of the disease. Therefore, piecewise incidence has been established in many pieces of literature [24, 28].

For many infectious diseases, it is important to consider the influences of delays on the disease dynamics. Time delay is very common in the transmission of infectious diseases and can be caused by various factors. The most notable reasons for a delay are the latency of the infection in a vector and the latent period in an infected host [9, 10, 23, 25]. In these cases, the infection takes some time in the infected host or vector and develops itself during a short period to become in the infection transmission stage. Many scholars have considered mathematical modeling of population dynamics with time delay [4, 12]. Introducing time delay can present more complex transmission dynamics properties.

In real life, prevention and control strategies may not be applied promptly, which means that the infection rate is related to the number of previously infected people. In this paper, we introduce the piecewise infection rate with delayed control strategies into the SIR epidemic model in order to describe the impact of control strategies on the development trend of infectious diseases. By studying the dynamic properties of the model, we show that control strategies can effectively reduce the number of infected individuals. To reduce fluctuations in the number of infected individuals, it is essential to ensure that control strategies are implemented in time.

The paper is organized as follows. The derivation and transformation of the epidemic model are presented in Section 2. We investigate the critical conditions for extinction and permanence of infectious diseases by Filippov system in Section 3. Section 4 contains dynamic properties analysis of the derived model without delay. In Section 5, we discuss the effect of delay on dynamic properties of the equilibria by the bifurcation theory. In Section 6, we make numerical simulations to verify the theoretical results. Our conclusions are given in Section 7 to end this work.

2. Model derivation

To illustrate our ideas, we begin with a simple compartment SI epidemic model. Assuming that the birth and death rate are zero (i.e., no demography), a simple SI model with saturation incidence is given by the following system of ordinary