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Analysis of Dynamic Properties of Forest Beetle Outbreak Model

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Abstract This paper mainly studies the dynamic properties of the forest beetle outbreak model. The existence of the positive equilibrium point and the local stability of the positive equilibrium point of the system are analyzed, and the relevant conclusions are drawn. After that, the existence of Turing instability, Hopf bifurcation and Turing-Hopf bifurcation are discussed respectively, and the necessary conditions for existence are given. Finally, the normal form of the Turing-Hopf point is calculated, and some dynamic properties at the point are analyzed by numerical simulation.

Keywords Reaction-diffusion equation, Turing instability, Hopf bifurcation, Turing-Hopf bifurcation

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1. Introduction

Disturbance is defined as any relatively discrete event that disrupts ecosystem, community or population structure in time and alters resources, substrate availability or physical environment [13]. Forest disturbance is considered to be a key factor affecting terrestrial biological and geochemical processes, and it is closely related not only to pests and diseases but also to forest fires. For this reason, many scholars have conducted many studies on the relationship between forests, beetles and forest fires.

Forest fires and pests are two natural disturbances to forests, and they have had a devastating impact on the succession of forests. Forest fires not only burn down forests and reduce its stand density, but also destroy forest structures and reduce the use value of forests. Forest fires kill seedlings and saplings, therefore prolong the forest regeneration period. The topsoil and rocks of the burned forest land are exposed, and many places become barren mountains and ridges, making it difficult for the forest to recover. According to statistics, an average of 100,000 square kilometers of forests in China are infested with pests and diseases every year. If it continues to develop, the shelterbelt project that involves half of the country's safety will be in danger of being destroyed by the pests population. In order to minimize the damage caused by pests and forest fires to the forest and to better carry out the forest management and control, many experts and scholars have established various pine beetle models for different environments and studied

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them (see [1–3, 7–9, 14, 15, 18–20, 26]). In [3], Chen proposed a mathematical model for beetle outbreaks as a single perturbation in forest population dynamics or in combination with wildfire perturbations:

$$\begin{cases} \frac{dV}{dt} = r_v V \left(1 - \frac{V}{K_v} - f_k \frac{B}{r+B}\right), \\ \frac{dB}{dt} = r_b B \left(1 - \frac{B}{K_e}\right) - \frac{\alpha B^2}{1+\beta B^2}, \end{cases} \tag{1.1}$$

where V and B represent the number of pine trees and the number of beetles at time t respectively. r_v and K_v are the natural growth rate and the carrying capacity of pine trees respectively. f_k represents the percentage of successfully attacked pines which are killed. r represents the threshold for the number of successfully attacked beetles. r_b and K_e are the natural growth rate and the carrying capacity of beetles respectively. α is the pine defense rate. β is the inverse of the beetle density, when the pine defense is saturated. All parameters involved with the model are positive.

Based on model (1.1), Chen proposed model (1.2) with forest fire disturbance as follows:

$$\begin{cases} \frac{dV}{dt} = r_v V \left(1 - \frac{V}{K_v} - f_k \frac{B}{r+B}\right) - P \frac{V}{K_v} M_v V, \\ \frac{dB}{dt} = r_b B \left(1 - \frac{B}{K_e}\right) - \frac{\alpha(1-cM_v)B^2}{1+\beta B^2} - M_b P \frac{V}{K_v} M_v B, \end{cases} \tag{1.2}$$

where M_v and M_b represent the impact intensity of a forest fire on trees and beetles, respectively. P represents the probability of a forest fire, and c is the parameter of fire weakening the pine tree's defense against beetles. M_v, M_b, P and c are positive parameters.

Since Lotka and Volterra proposed the predator-prey dynamic behavior model, many experts and scholars have studied various predator-prey models, which has laid a solid theoretical foundation for the latter study (see [4–6, 12, 16, 17, 21–24, 28–30]). This paper argues that beetles will gain certain benefits after successfully invading pine trees, and improves on the model proposed by Chen, changing the model from a competition model to a predator-prey model, considering that both pine trees and beetles can spread in space, thereby introducing a diffusion term, and establishing the following reaction-diffusion forest beetle outbreak model:

$$\begin{cases} \frac{\partial V(x,t)}{\partial t} = r_v V \left(1 - \frac{V}{K_v} - f_k \frac{B}{r+B} - P \frac{V}{K_v} M_v\right) + d_1 \Delta V, \\ \frac{\partial B(x,t)}{\partial t} = r_b B \left(1 - \frac{B}{K_e}\right) - \frac{\alpha(1-cM_v)B^2}{1+\beta B^2} - M_b P \frac{V}{K_v} M_v B + \xi V B + d_2 \Delta B, \\ V_x(0,t) = B_x(0,t) = 0, V_x(l\pi,t) = B_x(l\pi,t) = 0, \\ V(x,0) = V_0(x) \geq 0, B(x,0) = B_0(x) \geq 0, \\ x \in (0, l\pi), t > 0, \end{cases} \tag{1.3}$$

where $V(x,t)$ and $B(x,t)$ are the number of pine trees and the number of beetles at position x and time t respectively. $d_1 > 0$ and $d_2 > 0$ represent the diffusion coefficients of prey and predator respectively. $\xi > 0$ is the buff the beetles get, when the pine tree is attacked by beetles. The meanings of the remaining parameters are the same as those in (1.1) and (1.2), and will not be repeated here. The boundary condition is Neumann boundary condition and all parameters involved with the