Commun. Math. Res.
doi: 10.4208/cmr.2023-0014

Vol. **39**, No. 4, pp. 523-538
November 2023

Geometrical Characterizations of Non-Radiating Sources at Polyhedral and Conical Corners with Applications

Huaian Diao[1], Yueran Geng[1,*], Hongyu Liu[2] and Qinghua Yu[3]

[1] *School of Mathematics and Key Laboratory of Symbolic Computation and Knowledge Engineering of Ministry of Education, Jilin University, Changchun, Jilin 130012, P.R. China.*
[2] *Department of Mathematics, City University of Hong Kong, Kowloon, Hong Kong SAR, P.R. China.*
[3] *The School Attached to Dalian Maritime University, Dalian 116000, P.R. China.*

Received 16 March 2023; Accepted 18 July 2023

Abstract. Considering the acoustic source scattering problems, when the source is non-radiating/invisible, we investigate the geometrical characterization for the underlying sources at polyhedral and conical corner. It is revealed that the non-radiating source with Hölder continuous regularity must vanish at the corner. Using this kind of geometrical characterization of non-radiating sources, we establish local and global unique determination for a source with the polyhedral or corona shape support by a single far field measurement. Uniqueness by a single far field measurement constitutes of a long standing problem in inverse scattering problems.

AMS subject classifications: 35Q60, 378A46, 35P2

Key words: Non-radiating sources, corner singularity, vanishing, inverse scattering, single far-field measurement.

---

*Corresponding author. *Email addresses:* diao@jlu.edu.cn (H. Diao), gengyr1019@mails.jlu.edu.cn (Y. Geng), hongyliu@cityu.edu.hk (H. Liu), 3255742434@qq.com (Q. Yu)

1 Introduction

Let Ω be a bounded Lipschitz domain with a connected complement in $\mathbb{R}^3$. Consider the following time-harmonic acoustic source scattering problem:

$$\begin{aligned} &(\Delta+k^2)u(x)=f(x) \quad \text{in } \mathbb{R}^3, \\ &\lim_{r\to\infty} r\big(\partial_r - \mathrm{i}k\big)u=0, \quad r=|x|, \end{aligned} \tag{1.1}$$

where $f=\chi_\Omega\varphi$, $\varphi\in L^\infty(\mathbb{R}^3)$ and $k\in\mathbb{R}_+$. The limit in (1.1) is known as the Sommerfeld radiation condition which characterizes the outgoing nature of the radiating wave. Throughout this paper we assume that the wave number $k\in\mathbb{R}_+$ is fixed. By the variational approach, it is known that (1.1) admits a unique solution $u\in H^2_{loc}(\mathbb{R}^n)$ [11]. Therefore, the following asymptotic expansion for the acoustic wave field u to (1.1) is given by

$$u(x)=\frac{e^{\mathrm{i}k|x|}}{|x|}u_\infty(\hat{x})+\mathcal{O}\left(\frac{1}{|x|^{3/2}}\right) \quad \text{as} \quad |x|\to+\infty, \tag{1.2}$$

where $u_\infty(\hat{x})$ is referred to be the far field pattern of u and $\hat{x}=x/|x|$. By the Rellich theorem, there is one to one correspondence between the wave field u and the real analytic function $u_\infty(\hat{x})$ defined on the unit sphere $\mathbb{S}^2$.

In this paper we are mainly concerned with geometrical characterization of non-radiating source at polyhedral and conical corners. In the following we first give the definition of non-radiating source.

Definition 1.1. *We say that φ is a non-radiating source corresponding to* (1.1) *if the far field pattern of u to* (1.1) *associated with φ identically equals to zero, namely $u_\infty(x)\equiv 0$.*

It is clear that a non-radiating source φ is invisible to the far field measurement. By Rellich Theorem [11], if the invisibility of the source φ occurs, one directly has

$$\begin{cases} \Delta u+k^2u=\varphi & \text{in } \Omega, \\ u=\partial_\nu u=0 & \text{on } \partial\Omega, \end{cases} \tag{1.3}$$

where ν signifies the unit outward normal vector to $\partial\Omega$. We shall give geometrical characterizations of non-radiating sources at polyhedral and conical corners. Namely, we shall reveal that if Ω has a conic or polyhedral corner, where φ is Hölder continuous near it, then φ must vanish at the underlying corner. This kind of the geometrical characterization of non-radiating source can help us to study the inverse source shaper problem for (1.1), which can be described by

$$u_\infty(\hat{x}), \quad \hat{x}\in\mathbb{S}^2 \ \mapsto \ \partial\Omega, \tag{1.4}$$

which intends to determine the shape of the support of the inaccessible source f.