Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2023-0112

Vol. **34**, No. 5, pp. 1306-1331
November 2023

A Second-Order Implicit-Explicit Scheme for the Baroclinic-Barotropic Split System of Primitive Equations

Rihui Lan[1], Lili Ju[2,*], Zhu Wang[2] and Max Gunzburger[3]

[1] *School of Mathematical Sciences, Ocean University of China, Qingdao, Shandong 266100, China.*
[2] *Department of Mathematics, University of South Carolina, Columbia, SC 29208, USA.*
[3] *Oden Institute for Computational Engineering and Sciences, University of Texas at Austin, Austin, Texas 78712, USA.*

Received 15 April 2023; Accepted (in revised version) 1 July 2023

Abstract. The baroclinic-barotropic mode splitting technique is commonly employed in numerical solutions of the primitive equations for ocean modeling to deal with the multiple time scales of ocean dynamics. In this paper, a second-order implicit-explicit (IMEX) scheme is proposed to advance the baroclinic-barotropic split system. Specifically, the baroclinic mode and the layer thickness of fluid are evolved explicitly via the second-order strong stability preserving Runge-Kutta scheme, while the barotropic mode is advanced implicitly using the linearized Crank-Nicolson scheme. At each time step, the baroclinic velocity is first computed using an intermediate barotropic velocity. The barotropic velocity is then corrected by re-advancing the barotropic mode with an improved barotropic forcing. Finally, the layer thickness is updated by coupling the baroclinic and barotropic velocities together. In addition, numerical inconsistencies on the discretized sea surface height caused by the mode splitting are alleviated via a reconciliation process with carefully calculated flux deficits. Temporal truncation error is also analyzed to validate the second-order accuracy of the scheme. Finally, two benchmark tests from the MPAS-Ocean platform are conducted to numerically demonstrate the performance of the proposed IMEX scheme.

AMS subject classifications: 35Q86, 65M08, 65Y05

Key words: Primitive equations, baroclinic-barotropic splitting, implicit-explicit, strong stability preserving RK, SSH reconciliation.

*Corresponding author. *Email addresses:* `lanrihui@ouc.edu.cn` (R. Lan), `ju@math.sc.edu` (L. Ju), `wangzhu@math.sc.edu` (Z. Wang), `max.gunzburger@austin.utexas.edu` (M. Gunzburger)

1 Introduction

The ocean is a critical component of the climate and plays a significant role in our daily lives. To comprehend and simulate ocean dynamics, various computational models have been developed based on the fundamental laws of physics and the characteristics of geophysical flows. The primitive equations [1–3], a simplification of the Navier-Stokes equations, are one of these models. They have been widely used in real-world applications and coupled with tracers such as temperature, salinity, and chemicals to fluid velocity, fluid depth, and pressure. Due to the stratification effect of the ocean [4], the fluid is often modeled as a stack of immiscible layers, each with a uniform fluid density. Such layered models simplify the computational costs of stratified fluid flows and perform well at portraying vertical profiles. For additional details, readers can refer to [3, 4] and the references cited therein. Nonetheless, conducting long-term numerical simulations for layered models is still quite challenging because of their extensive computational complexity, which arises from the integration of the large-scale oceanic system. Specifically, the oceanic dynamics frequently consist of various time scales resulting from the interplay between external and internal gravity waves and the rotation of the Earth. In layered ocean models, the barotropic mode is considered as the fastest of the entire spectrum of inertial-gravity waves, while the remaining part is referred to as the baroclinic mode. The barotropic mode has a wave speed of up to approximately 200 m/s, while the advection-dominated baroclinic mode is only about 2 m/s [5]. Since the barotropic velocity is about two orders of magnitude faster than the baroclinic one, the barotropic mode highly restricts the time step size under the Courant-Friedrichs-Lewy (CFL) condition if the entire system is advanced via an explicit uniform stepping scheme. Therefore, it is natural to split these two modes in ocean dynamics and evolve them separately over time.

The barotropic-baroclinic mode splitting technique has been extensively studied in the literature [6–14]. This approach splits the primitive equations into two modes: the barotropic mode, which is obtained through vertical averaging and is a 2D mode, and the baroclinic mode, which is the difference between the original velocity and the barotropic one and is a 3D mode. Many modern ocean models, such as the "MPAS-Ocean" [15], which was developed by Los Alamos National Laboratory and collaborating institutions, use this mode splitting process to design efficient numerical methods for solving the primitive equations. To advance the baroclinic mode, these models use explicit time integration schemes that take advantage of the natural parallelism and ease implementation of explicit stepping. However, there are two methods for integrating the barotropic mode: an explicit-subcycling approach and a semi-implicit solver approach. The explicit-subcycling approach advances the barotropic mode using a smaller time step size to meet the more restrictive CFL condition for the barotropic subsystem [11,13,15]. The resulting numerical scheme, known as the split-explicit (SE) method, requires updating the intermediate values during each subcycling step. This approach imposes intensive message exchange among the processors/cores of high-performance computing clusters, which may significantly affect the parallel scalability. Moreover, the SE method is not automati-