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Approximation Properties of Newman Type Interpolation Rational Functions with Fewer Nodes

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Abstract. In the present note, we consider the problem: how many interpolation nodes can be deleted from the Newman-type rational function such that the convergence rate still achieve.

Key Words: Rational function approximation, Newman-type rational approximation, interpolation nodes, convergence rate.

AMS Subject Classifications: 41A17

1 Introduction

Let P_n denote the set of all algebraic polynomials of degree at most $n, n \geq 0$ and let R_n be the class of all rational functions:

$$r = \frac{p}{q}, \quad p, q \in P_n, \quad q \neq 0.$$

For any $f \in C_{[-1,1]}$, we denote by

$$E_n(f) = \inf_{p \in P_n} ||f - p||_{[-1,1]}, \quad R_n(f) = \inf_{r \in R_n} ||f - r||_{[-1,1]},$$

the errors in best approximation of f on $[-1, 1]$ by elements of P_n and R_n, respectively. Here and in what follows, $|| \cdot ||$ stands for the uniform norm on an indicated interval.

In the following, we denote by c positive constant (different each time, in general) that is absolute or depends on parameters not essential for the argument. If $A(k, n, x, \cdots)$ and $B(k, n, x, \cdots)$ are positive real numbers depending on parameters $k, n, x, \cdots$, then the notation

$$A(k, n, x, \cdots) = O(B(k, n, x, \cdots))$$

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means that there exists positive real number c independent of $k,n,x,\cdots$, such that

$$A(k,n,x,\cdots)\le cB(k,n,x,\cdots).$$

The notation

$$A(k,n,x,\cdots)\sim B(k,n,x,\cdots)$$

means that there exist c_1, c_2 independent of $k,n,x,\cdots$, such that

$$c_1B(k,n,x,\cdots)\le A(k,n,x,\cdots)\le c_2B(k,n,x,\cdots).$$

Let

$$X_n=\{x_k^{(n)}:k=1,2,\cdots,n,\ 0<x_n^{(n)}<x_{n-1}^{(n)}<\cdots<x_1^{(n)}\le 1\}$$

be a set of n distinct points in $(0,1]$, and let

$$P_n=\prod_{k=1}^{n}(x+x_k^{(n)}), \tag{1.1}$$

(in the sequence, when there is no confusion, the superscript (n) will be omitted).

The Newman-type rational interpolation to $|x|$ (see [3]) at the set of the points

$$\{-x_1,\cdots,-x_{n-1},-x_n,0,x_n,x_{n-1},\cdots,x_1\} \tag{1.2}$$

is defined by

$$r_n=r_n(X_n;x)=x\frac{p_n(x)-p_n(-x)}{p_n(x)+p_n(-x)}. \tag{1.3}$$

Since $r_n(X_n;x)$ as well as $|x|$ are even functions, the approximation error

$$e_n(X_n;x)=||x|-r_n(X_n;x)| \tag{1.4}$$

may be restricted to the interval $[0,1]$, where it can be represented in the form

$$e_n(X_n;x)=\left|\frac{2xh_n(X_n;x)}{1+h_n(X_n;x)}\right|,\quad x\in[0,1], \tag{1.5}$$

where

$$h_n(X_n;x)=\frac{p_n(-x)}{p_n(x)}=\prod_{k=1}^{n}\frac{-x+x_k}{x+x_k}. \tag{1.6}$$

The well-known result of S.Bernstion [1] is that

$$E_n(|x|)\sim\frac{1}{n}.$$