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PASSAGE OF HIGH-FREQUENCY SIGNALS THROUGH POWER TRANSFORMERS.

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Abstract. We consider the possibility of passing high-frequency signals past power transformers forming part of an electrical grid. We first model a transformer, including its laminated core, to obtain asymptotic behaviour of currents and voltages in the secondary circuit. Having got this we are able to determine the effects of different by-pass mechanisms which might be tried to get the high-frequency signal from the primary to the secondary circuit.

Key words. Homogenisation, thin layers, high frequency.

1. Introduction

A possible cheap method – hopefully requiring little extra hardware and wiring – of transferring data between, on the one hand, individual consumers of electricity, and on the other, one central data-processing/interrogation point, is to transmit a high-frequency signal along power lines. Unfortunately, there are a number of barriers to such a signal, in the form of transformers used to step down the voltage of the power supply. The transformers used for reducing the voltages utilised by the main transmission lines are not a major problem, as there are relatively few of them and installing special equipment to get high-frequency signals past them might be commercially viable. However, there are many more transformers employed in local sub-stations so a cheap and simple way of ensuring signals get past these is needed, if the technique is to be economical.

We start, in Section 2, by looking at basic electromagnetic theory applying in a power or distribution transformer. In particular, we see one reason why it is observed, [3], [9], [10], [22], that at "low" frequencies it is seen that power loss by the transformer (power input into the primary winding less power output by the secondary coil) is small, order of frequency squared, while for "high" frequencies there is higher power loss, of order one, and power output decaying as a power of frequency. To get the correct sizes of losses, the laminated structure of the transformer's magnetic core must be modelled. Typically, the core consists of alternating layers of a conducting ferro-magnetic and an insulating non-magnetic material. The width of the layers is order 0.4 mm, compared with a macroscopic length scale of 1 m. This means that averaged equations can be derived. In building our model we take, for simplicity, the ferro-magnetic core to behave as a simple material so that magnetic induction is proportional to magnetic field; non-linear behaviour, such as saturation, hysteresis and kinetic effects are all disregarded. Note that power losses discussed in the present paper result largely from eddy currents alone. (It should be noted, however, that in practice hysteresis produces most of the power losses, [18].) We see that the laminated structure of the core keeps (as is well known) power losses due to eddy currents low at a normal mains frequency of 50 Hz, but there is a large power loss at the desired frequency of the signal.

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The main treatment of the laminated core is based on homogenisation, using the method of multiple-scales (see, for example, [8]), to obtain an averaged model for this particular structure. This particular problem seems not to have been fully solved in the substantial literature on homogenisation, much of it rigorous, see [19] and [20] for quite general problems, and [1], [2], [4], [5], [6], [11], [12], [17], [21] which consider electro-magnetic fields in various types of heterogeneous media. Of particular note are [13], which looks at layered materials, one of which is a perfect insulator, and [7], which discusses the relationship between the small size of included materials and other small parameters which can appear in particular problems. In the case of present interest, we shall be concerned with the balances between small layer size and high frequency of the electric currents. Other limiting parameters which arise briefly in this work are the high ratios of electrical conductivities and of magnetic permeabilities between the insulating and the ferro-magnetic materials.

Section 2 re-derives, using the method of multiple scales, the fast and slow spatial dependencies of the electromagnetic field, found in [13], in a distinguished limit of thin layers and high ratio of conductivities. Extending what has been done previously done in the literature, we then use these results to obtain the inductances for transformers for various limiting cases of interest.

In Section 3 we use the results of the internal modelling in considering the current flow when a power supply is connected to the primary coil and a load is connected to the secondary. It is clear that, without any extra device linking the two sides of the transformer, there is negligible transmission of any high-frequency electromagnetic signal across the transformer. Connecting some sort of impedance (in the simplest cases, just a resistor, capacitor or inductor) across the transformer to link the primary and secondary circuits in such a way as not to change the performance at low, mains, frequencies, is seen not to significantly enhance the transmission of high frequencies.

The possible changed internal behaviour of the transformer windings at high frequencies is briefly looked at in the Discussion, Section 4.

2. Modelling the Magnetic Core

2.1. Basic Case. We start by considering a single piece of iron or steel, surrounded by an air (or other insulating) gap, which in turn is surrounded by a layer carrying an electric current. This surface current density represents the current being carried by the wires in the coil. For simplicity a two-dimensional situation, as in Fig. 1, is taken. With this simplified geometry, the electric field lies in the $x_1 - x_2$ plane, while the magnetic field is in the normal, x_3, direction. We can then write $\boldsymbol{E} = (E_1, E_2, 0) = \boldsymbol{E}(x_1, x_2, t)$ for the electric field and represent the magnetic induction $\boldsymbol{B} = (0, 0, B)$ by the scalar quantity $B(x_1, x_2, t)$. Maxwell's equations,

$$(2.1) \qquad \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}\,, \quad \nabla \times \left(\frac{\boldsymbol{B}}{\mu}\right) = \epsilon \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{j}\,, \quad \nabla\cdot\boldsymbol{E} = \frac{\rho}{\epsilon}\,, \quad \nabla\cdot\boldsymbol{B} = 0\,,$$

then reduce to

$$(2.2) \qquad \frac{\partial E_1}{\partial x_2} - \frac{\partial E_2}{\partial x_1} = \frac{\partial B}{\partial t}\,, \quad \frac{\partial}{\partial x_2}\left(\frac{B}{\mu}\right) = j_1\,, \quad \frac{\partial}{\partial x_1}\left(\frac{B}{\mu}\right) = -j_2\,,$$

as the last of (2.1) automatically holds, the third is not used, and we can neglect the $\epsilon\, \partial \boldsymbol{E}/\partial t$ term for frequencies much less than $1/(w\sqrt{\epsilon_0 \mu_0}) = \mathrm{O}(3 \times 10^8\,\mathrm{s}^{-1})$. Note