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A NEW CURVATURE-BASED IMAGE REGISTRATION MODEL AND ITS FAST ALGORITHM

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Abstract. Recently, Chumchob-Chen-Brito (2011) proposed the so-called mean curvature model which appeared to deliver better registration results for both smooth and non-smooth deformation fields than a large class of competing methods. However, the two displacement variables in a deformation field are regularized separately in their model and so coupling between them is not present. Therefore their mean curvature model has a weakness and may not yield the best registration results in some situations such as in non-smooth registration problems with non-axis-aligned discontinuities, as expected of a high order model. To design a new model based on interdependence between components of the deformation field, suitable for smooth and non-smooth registration problems, we propose a new vectorial curvature regularizer in this paper and present an iterative method for numerical solution of the resulting variational model. Experiments using both synthetic and realistic images confirm that the proposed model is more robust than the Chumchob-Chen-Brito (2011) model in registration quality for a wide range of examples.

Key words. Deformable registration, regularization, multilevel, curvature.

1. Introduction

In image processing, one is often interested not only in analyzing an image but also in comparing images in order to combine information or track changes. For this reason, image registration, also called image matching or image warping, is one of the most useful and challenging tasks in imaging applications. It is a problem frequently encountered in diverse application areas, such as astronomy, art, biology, chemistry, remote sensing and so on. Especially, in medical applications, noninvasive imaging is increasingly used in almost all stages of patient care: from disease detection to treatment guidance and monitoring. For comprehensive surveys of these applications, we refer to [8, 25], and references therein.

A general framework of image registration can be stated as follows: given two images of the same object, respectively the reference R (fixed) and the template T (moving), we search for a vector-valued transformation $\boldsymbol{\varphi}$ defined by

$$\boldsymbol{\varphi}(\boldsymbol{u})(\cdot) : \mathbb{R}^d \to \mathbb{R}^d, \ \boldsymbol{\varphi}(\boldsymbol{u})(\boldsymbol{x}) : \boldsymbol{x} \mapsto \boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})$$

or equivalently the unknown displacement field $\boldsymbol{u}$

$$\boldsymbol{u} : \mathbb{R}^d \to \mathbb{R}^d, \ \boldsymbol{u} : \boldsymbol{x} \mapsto \boldsymbol{u}(x) = (u_1(\boldsymbol{x}), u_2(\boldsymbol{x}), \cdots, u_d(\boldsymbol{x}))^\top,$$

such that the transformed template $T \circ \boldsymbol{\varphi}(\boldsymbol{u}(x)) = T(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})) = T(\boldsymbol{u})$ becomes similar to the reference R. Here $d \in \mathbb{N}$ represents the spatial dimension of the images which is usually $d = 2$ in the two-dimensional case or $d = 3$ the three-dimensional case with boundary $\partial\Omega$. Without loss of generality, we focus on $d = 2$ and state that registration results are readily extendable to $d = 3$. Then $\boldsymbol{x} = (x_1, x_2)$ and $d\Omega = dx_1 dx_2$. When the corresponding location $\boldsymbol{\varphi}(\boldsymbol{u}(\boldsymbol{x})) = \boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})$ is calculated for each spatial location $\boldsymbol{x}$ in the image domain $\Omega \subset \mathbb{R}^2$, an image interpolation is required to assign the image intensity values for the transformed template $T(\boldsymbol{u})$

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at non-grid locations within image boundaries. For location outside the image boundaries, the image intensities are usually set to be a constant value, typically zero, we can refer to [25]. It is well worth noticing that the displacement $\boldsymbol{u}$ is more intuitive than the transformation $\boldsymbol{\varphi}$ because it can measure how much a point in the transformed template $T(\boldsymbol{u})$ has moved away from its original position in T. Here we shall restrict ourselves to scalar or gray intensity images and model them as compactly supported functions mapping from the image domain $\Omega \subset \mathbb{R}^2$ into $V \subset \mathbb{R}_0^+$. Assuming the image intensities of R and T are comparable, the image registration problem can be formulated as the following similarity minimization

$$\min_{\mathbf{u}} \Big\{ \mathcal{D}(\boldsymbol{u}) = \frac{1}{2} \int_\Omega (T(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})) - R(\boldsymbol{x}))^2 d\Omega \Big\}. \tag{1}$$

It has long been known that image registration (or the above model) is an ill-conditioned problem [25]. As a consequence, regularization is inevitable. A common treatment of the registration problem is based on the Tikhonov type regularization approach: find the transformation $\boldsymbol{u}$ by minimizing the joint energy functional

$$\mathcal{J}_\alpha(\boldsymbol{u}) = \mathcal{D}(\boldsymbol{u}) + \alpha \mathcal{R}(\boldsymbol{u}) \tag{2}$$

where $\mathcal{R}(\boldsymbol{u})$ is a deformation regularizer and $\alpha > 0$ is the regularization parameter that compromises similarity and regularity.

Non-surprisingly, the choice of $\mathcal{R}(\boldsymbol{u})$ is very crucial for effective registration. For instance, $\mathcal{R}(\boldsymbol{u}) = \|\boldsymbol{u}\|_2$ is sufficient to ensure (2) to be well defined but it does not lead to a useful model. There exist many models for deformable image registration, mainly differing in how regularization is introduced. For smooth registration problems (where $\boldsymbol{u}$ can be assumed to be smooth), the common regularization techniques such as diffusion-, elastic-, or curvature-based image registration are known to generate globally smooth deformation fields, see more details in [11, 13, 14, 24, 23, 22, 25, 30] and references therein. However, these techniques become poor if discontinuities or steep gradients in the deformation fields are required. The total variation-based (TV) image registration is better for preserving discontinuities of the deformation fields [15, 16, 28]. Nevertheless, the TV model may not be appropriate for smooth registration problems. Clearly $\boldsymbol{u}$ is unknown and any assumption on it is practically unrealistic; therefore a common wish is for a model to achieve robustness regardless of what properties $\boldsymbol{u}$ may have.

Recently, Chumchob-Chen-Brito [12] proposed the so-called mean curvature-based variational model

$$\min_{\boldsymbol{u}} \mathcal{J}_\alpha(\boldsymbol{u}) = \mathcal{D}(\boldsymbol{u}) + \alpha \mathcal{R}^{\text{CCBc}}(\boldsymbol{u}), \qquad \mathcal{R}^{\text{CCBc}}(\boldsymbol{u}) = \sum_{l=1}^{2} \int_\Omega \phi(\kappa_{\text{CCB}}(u_l)) d\Omega. \tag{3}$$

Here $\phi(s) = \frac{1}{2}s^2$ and $\kappa_{\text{CCB}}(u_l) = \nabla \cdot \frac{\nabla u_l}{|\nabla u_l|_\beta}$ with $|\nabla u_l|_\beta = \sqrt{((u_l)_x)^2 + ((u_l)_y)^2 + \beta}$, and $\beta > 0$ is a small real parameter for avoiding non-differentiability where $|\nabla u_l| = 0$ [12, 15, 16, 28]. The work of [12] showed that $\mathcal{R}^{\text{CCBc}}$ shares some attractive properties with the Fischer-Modersitzki's linear curvature-based regularizer [14, 24, 25], e.g. the energy functional (3) does not penalize affine-linear transformations and it is invariant under planar rotation and translation. Moreover, model (3) appeared to deliver excellent results for both smooth and non-smooth registration problems, provided that u_1 and u_2 are not strongly coupled. This is because the nonlinear diffusion processes resulting from the first variation of $\mathcal{R}^{\text{CCBc}}(\boldsymbol{u})$ do not enforce coupling between the primary components of the deformation field, u_1 and u_2. Thus their mean curvature model may not obtain a good registration in some