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SECOND-ORDER TWO-SCALE ANALYSIS METHOD FOR DYNAMIC THERMO-MECHANICAL PROBLEMS IN PERIODIC STRUCTURE

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Abstract. In this paper, we develop the second-order two-scale (SOTS) analysis method and numerical algorithm for dynamic thermo-mechanical problems of composite materials with 3-D periodic configuration. In the problem considered, there exists a mutual interaction between the displacement and temperature fields. By the asymptotic expansion of temperature and displacement fields, the cell problems, effective thermal and mechanical parameters, homogenized equations and SOTS formulas of temperatures and displacements are obtained. The numerical algorithm based on the SOTS method is given. Finally, some numerical examples are shown. The numerical results show that the SOTS method is feasible and valid to predict the dynamic thermo-mechanical behaviors of periodic composite materials.

Key words. Composites with 3-D periodic configuration, dynamic thermo-mechanical problems, the SOTS analysis method, numerical algorithm.

1. Introduction

With the rapid advance of materials science and technology, composite materials have been widely used to a variety of industrial fields owing to the advantageous physical and mechanical properties. With the appearance of complex and extreme service environments, composite structures usually work under multi-physical fields coupled circumstances. And it is important to understand the thermo-mechanical responses of them in engineering applications. Up to now, some research has been performed on thermo-mechanical problems of periodic composites. However, some studies were devoted to one-way thermo-mechanical coupling problems [1-4], namely, the thermal effects affect the mechanical filed but not vice versa. Other studies have focused on developing different types of micromechanical models with simplified microstructures [5-8] and various numerical modeling approaches [9-11] to obtain the effective thermal and mechanical properties or homogenized behaviors. But in many engineering applications, the understanding of the local fluctuation of temperature and displacement fields is much more important. Besides, in some situations, such as the thermal shock phenomena, the dynamic thermo-mechanical problem should be considered. And the fully coupled analysis will lead to more accurate results. So it is significant and meaningful to study dynamic thermo-mechanical problems of periodic composites. In this field, Francfort [12] and Parnell [13] have given the homogenized procedure for the dynamic problems with different periodic configurations.

The dynamic thermo-mechanical problem is strongly coupled by hyperbolic and parabolic equations, so the transient displacement and temperature fields must be

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solved simultaneously. Thus, it is difficult to apply analytical methods to study it. As for numerical solutions, due to the sharply varying of material coefficients, in order to capture the local fluctuation behaviors of temperature and displacement fields and their derivatives, the mesh size must be very small while employing the traditional finite element method (FEM) and finite difference method (FDM). So it leads to tremendous amount of computer memory and CPU time. It is needed to develop new effective method for predicting the physical and mechanical performances.

Based on the homogenization methods [14-16], various multi-scale methods have been proposed [22, 23]. They only considered the first-order asymptotic expansions. In recent years, Cui et al. introduced the Second-Order Two-Scale (SOTS) analysis method [17-19] to predict the physical and mechanical behaviors of composites. By the second-order correctors, the microscopic fluctuation of physical and mechanical behaviors inside the materials can be captured more accurately. Feng et al. [20] studied the two-scale analysis for the static thermo-mechanical coupling problem of periodic composites. After that, Wan [21] studied the dynamic thermo-mechanical problem by two-scale analysis method and gave the numerical results of 1-D. In this paper, we study the SOTS's numerical method on dynamic thermo-mechanical problems of 3-D periodic composites (Fig.1) which is much more popular in engineering practice. In 3-D case, temperatures, displacements, temperature gradients and stresses are calculated.

The reminder of this paper is outlined as follows. In section 2, the SOTS asymptotic analysis for the dynamic thermo-mechanical problem is presented briefly. Section 3 describes the algorithm procedure. And some numerical results are shown in section 4. Finally the conclusions are given.

For convenience, we use the Einstein summation convention on repeated indices throughout the paper. For simplicity, we do not give the definitions of the associated Sobolev spaces in this paper, and we refer the reader to some classical books [26, 27].

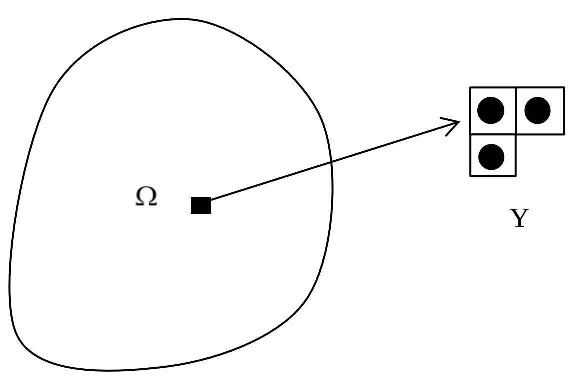

Figure 1. Macroscopic and Microscopic structure.