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AN UPPER ESTIMATE OF SOLUTION FOR A GENERAL CLASS OF PARABOLIC EQUATIONS*

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Abstract The upper estimates of the functions that satisfy some differential-integral inequality are established in this paper. We obtain the uniform estimates of maximum of the solutions for a general class of parabolic equations and extend some known results.

Key Words Differential-integral inequalities; upper estimate; parabolic equation.

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1. Introduction

In [1,2], O.A. Ladyzhenskaya and N.N. Ural'tseva, etc., introduced the natural construction conditions for elliptic and parabolic equations with principal part of divergence form. Based on the essential works of E. DeGiorgi [3], they obtained the solutions of the boundary value problem of elliptic and parabolic equation that belong to the DeGiorgi function classes and showed that the function in the DeGiorgi classes are Hölder continuous. In [4,5], Moser then established the Harnack inequality for the parabolic equations using different methods, and obtained the Hölder continuity for the solution of the parabolic equation. Later, E.DiBenedetto and N.S. Trudinger, in [6] proved that the function in the DeGiorgi elliptic type class satisfies the Harnack inequality, then G.L. Wang, in [7] obtained the same results for the DeGiorgi parabolic type function class and extended the results in [2,5]. There is the wide literature to study the regularity and existence of the solutions for the elliptic and parabolic equations under the natural construction conditions, for the later works in this fields, one can see the references [8–10].

G.M. Lieberman (see [11–14]) recently obtained many beautiful results for the maximum estimates and regularity of the solutions for the elliptic and parabolic equations.

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In [11], he had a general extension for the natural construction conditions of elliptic equation and considered the corresponding problem for parabolic equation. In [14], Lieberman obtained the maximum estimates for solutions of degenerate parabolic equations in divergence form. Meantime, A.G.Korolev, in [15], considered the boundedness of generalized solution of the elliptic differential equation with nonpower nonlinearities using Orlicz space theory. In [10], G.C.Dong discussed the following first boundary value problem of parabolic equation:

$$\begin{cases} u_t - \sum \dfrac{d}{dx_i} a_i(x,t,u,u_x) + a(x,t,u,u_x) = 0 \\ u|_S = \psi|_S \\ u|_{t=0} = \phi(x) \end{cases} \tag{1}$$

with the following construction conditions:

$$\begin{gathered} \nu(|u|)|\xi|^2 \le \sum \frac{\partial a_i}{\partial p_j}\xi_i\xi_j \le \mu(|u|)|\xi|^2, \quad \forall \xi \in R^n \\ \sum\left(|a_i| + \left|\frac{\partial a_i}{\partial u}\right|\right)(|p|+1) + \sum\left|\frac{\partial a_i}{\partial p_j}\right|\left(|p|^2+1\right) \\ + |a| + \sum\left|\frac{\partial a_i}{\partial x_j}\right| \le \mu(|u|)(1+|p|^2) \end{gathered} \tag{2}$$

where ν and μ are the functions given, S is the parabolic boundary of domain.

There are also some results for the existence and regularity of solutions of the problem (1) under the following construction conditions:

$$\begin{cases} \sum \dfrac{\partial a_i}{\partial p_j}\xi_i\xi_j \ge \nu(|u|)\left(1+|p|^{m-2}\right)|\xi|^2, \quad \forall \xi \in R^n \\ \sum\left(|a_i| + \left|\dfrac{\partial a_i}{\partial u}\right|\right)(|p|+1) + \sum\left|\dfrac{\partial a_i}{\partial p_j}\right|(|p|^2+1) \\ \qquad +|a| + \sum\left|\dfrac{\partial a_i}{\partial x_j}\right| \le \mu(|u|)(1+|p|^m), \quad (m>2) \end{cases}$$

Moreover, in order to obtain the estimate of maximum of the solution of (1), there is the following condition (see [10]):

$$u\left[-\sum \frac{\partial a_i(x,t,u,0)}{\partial x_i} + a(x,t,u,0)\right] \ge b_1u^2 - b_2, \quad b_1, b_2 \ge 0 \tag{3}$$

there is a similar assumption for elliptic equation (see [1,8,9]).

We attempt, in this paper, to generalize the conditions (2) and (3), and discuss the maximum estimate of the solution of (1) under general hypotheses, i.e.,

$$\begin{cases} \sum \dfrac{\partial a_i}{\partial p_j}\xi_i\xi_j \ge \nu(|u|)h(|p|)|p|^{-1}|\xi|^2 - C \\ \sum\left(|a_i| + \left|\dfrac{\partial a_i}{\partial u}\right|\right)(|p|+1) + \sum\left|\dfrac{\partial a_i}{\partial p_j}\right|(|p|^2+1) \\ \qquad +|a| + \sum\left|\dfrac{\partial a_i}{\partial x_j}\right| \le \mu(|u|)(1+|p|h(|p|)) \end{cases} \tag{4}$$