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THE ATTRACTORS FOR LANDAU-LIFSHITZ-MAXWELL EQUATIONS

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Abstract The existence of the attractors of the periodic initial value problem for the Landau-Lifshitz-Maxwell equations in one and two space dimensions is proved. We also get accurate estimates of the upper bounds of Hausdorff and fractal dimensions for the attractors by means of uniform *a priori* estimates for time and Lyapunov functional method.
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1. Introduction

In 1935, Landau Lifshitz [1] proposed the following coupled system of the nonlinear evolution equation

$$Z_t = \lambda_1 Z \times (\triangle Z + H) - \lambda_2 Z \times (Z \times (\triangle Z + H)) \tag{1.1}$$

$$\nabla \times H = \frac{\partial E}{\partial t} + \sigma E \tag{1.2}$$

$$\nabla \times E = -\frac{\partial H}{\partial t} - \beta \frac{\partial Z}{\partial t} \tag{1.3}$$

$$\nabla \cdot H + \beta \nabla \cdot Z = 0,\ \nabla \cdot E = 0 \tag{1.4}$$

where $\lambda_1, \lambda_2, \sigma, \beta$ are constants, $\lambda_2 \geq 0, \sigma \geq 0$, the unknown vector-valued function $Z(x,t) = (Z_1(x,t), Z_2(x,t), Z_3(x,t))$ denotes the microscopic magnetization field, $H(x,t) = (H_1(x,t), H_2(x,t), H_3(x,t))$ the magnetic field, $E(x,t) = (E_1(x,t), E_2(x,t), E_3(x,t))$ the electric field, $H^e = \triangle Z + H$ the effective magnetic field, $\triangle = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$, $\nabla = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \cdots, \frac{\partial}{\partial x_n}\right)$, "$\times$" the cross product of the vector in R^3.

If $H = 0, E = 0$, we obtain the Landau-Lifshitz system with Gilbert term

$$Z_t = \lambda_1 Z \times \triangle Z - \lambda_2 Z \times (Z \times \triangle Z) \tag{1.5}$$

where $\lambda_2 > 0$ is a Gilbert damping coefficient. In [2–4], the properties of the solution for the system of the equation (1.5) and the closed new links between the solution and the harmonic map on the compact Riemann manifold have been studied extensively. When $\lambda_2 = 0$, the system of the equation (1.5) becomes

$$Z_t = \lambda_1 Z \times \triangle Z \tag{1.6}$$

In the case of $n = 1$, it is an integral system, and has soliton solutions. In [5–13], the authors have studied in detail the solitons for (1.6) and the interaction among solitons, and the infinite conservative laws, and the inverse scattering method, and the relation with the nonlinear Schrödinger equations. As pointed out in [14], the system of the equation (1.6) is a strongly degenerate quasilinear parabolic system. In [14–22], we have investigated extensively the classic and generalized solutions to the initial value problem and other kinds of boundary value problem for the system of the equation (1.6), and some properties of the solutions, and further obtained the global generalized solutions for $n \geq 2$.

The purpose of this paper is to study the following periodic initial value problem for the system (1.1)–(1.4)

$$Z(x + 2De_i, t) = Z(x, t),\ H(x + 2De_i, t) = H(x, t)$$

$$E(x + 2De_i, t) = E(x, t),\ (x \in \Omega, t \geq 0, i = 1, 2, \cdots, n) \tag{1.7}$$

$$Z(x, 0) = Z_0(x),\ H(x, 0) = H_0(x),\ E(x, 0) = E_0(x),\ (x \in \Omega) \tag{1.8}$$

where $x + 2De_i = (x_1, \cdots, x_{i-1}, x_i + 2D, x_{i+1}, \cdots, x_n), (i = 1, 2, \cdots, n), D > 0, \Omega \subset R^n$ is n-dimensional cube with width $2D$.

In Section 2 we give the *a priori* estimate uniformly with respect to the time t for the smooth solution of (1.1)–(1.4), (1.7), (1.8). In Section 3 the existence of the attractor is proved. Finally the estimate of the dimension for the attractor is obtained in Section 4.

2. Priori Estimates

For the sake of simplicity denote $\|\cdot\|_{L_p} = \|\cdot\|_p, p \geq 2$.

Lemma 2.1 *Assume* $|Z_0(x)| = 1$. *Then for the smooth solution of the periodic initial value problem* (1.1)–(1.4), (1.7), (1.8) *there are*

$$|Z(x, t)| = 1,\ x \in \Omega, t \geq 0 \tag{2.1}$$

Proof Making the scalar product of Z with (1.1), we get

$$\frac{\partial}{\partial t}|Z(x, t)|^2 = 0$$

Then the conclusion of the lemma is proved.