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L^p ESTIMATES FOR A CLASS OF INTEGRAL OPERATORS*

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Dedicated to Professor Gu Chaohao on the occasion of his 70th birthday and his 50th year of educational work

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Abstract With the aid of the nonhomogeneous Calderon-Zygmund decomposition the present paper studies the L^p estimates for a class of integral operators related to the boundary value problems of degenerate elliptic equations. The method used here is also applicable to general evolution equations of elliptic type.

Key Words Degenerate elliptic; boundary value problem; L^p-estimation.
Classification 35J70, 35F10.

0. Introduction

Since Calderon and Zygmund initiated the theory on singular integral operators, the study about L^p boundedness of integral operators has been systematically developed. A simple and straightforward treatment for such operators can be found in [1] [2]. However, when studying evolution equations and degenerate elliptic equations we shall encounter that their resolvents are some integral operators with symbols spatially inhomogeneous. Generally speaking, in some distinguish direction (the dual of the time variable or the degenerate variable) the symbol has a different measure from those of the other and indeed the operator considered is not standard pseudodifferential operator in Hormander sense but can be expressed in terms of an integral operator with respect to one parameter family of standard pseudodifferential operators or singular integral operators. To the author's knowledge there has been no systematic discussions on the L^p boundedness for such operators. This is the motivation of the present paper. In attacking this problem the difficulty we run across is the inhomogeneity of their

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symbols. We have to make some modification of Calderon-Zygmund decomposition and relevant techniques.

In Section 1 a general frame for the study of L^p boundedness of the operators mentioned above is given.

In Section 2 as a preparation, a boundary value problem for an ordinary differential equation is studied. In Section 3, as an application of the result in Section 1, the L^p estimates of solutions to the Dirichlet problem for a degenerate elliptic equation are discussed. And a modification of Calderon-Zygmund decomposition is presented. Maybe, this is of independent interests. The result obtained is the generalization of [3].

In the appendix as another application, the L^p estimates of solutions to an elliptic evolution equation as well as an interpolation theorem about L^p estimation are obtained. The method used in this section is also applicable for general evolution equations.

1. L^p Boundedness of a Class of Integral Operators

This section concerns with the L^p boundedness of a class of integral operators involving one parameter family of convolution operators. If $K(y,\sigma,x) : \mathbf{R}^1 \ni y \to S'(\mathbf{R}^1 \times \mathbf{R}^n)$, then for each $f(\sigma,x) \in C_c^\infty(\mathbf{R}^{n+1})$ we can define an operator

$$Kf = \int K(y,\sigma,\cdot) * f(\sigma,\cdot) d\sigma \tag{1.1}$$

This is an integral operator involving convolution in x. If $K = K(x) \in S'$, $f \in C_c^\infty(\mathbf{R}^n)$, the boundedness of $K * f$ in $L^p(\mathbf{R}^n)$ is discussed in [1] [2]. In the present paper we shall study the L^p boundedness of (1.1). First of all we have

Lemma 1.1 *Suppose that* $K(y,\sigma,x) : \mathbf{R}^1 \ni y \to S'(\mathbf{R}^1 \times \mathbf{R}^n)$, $\hat{K}(y,\sigma,\xi) : \mathbf{R}^1 \times \mathbf{R}^1 \ni (y,\sigma) \to \hat{K} \in C^\infty(\mathbf{R}^n \backslash 0) \cap C(\mathbf{R}^n)$ *and for any* $\xi \in \mathbf{R}^n$, $\hat{K}$ *is piecewisely continuous in* y, σ. *Moreover,*

$$\sup_{y,\xi} \int |\hat{K}| d\sigma \le C \tag{1.2}$$

$$\sup_{\sigma,\xi} \int |\hat{K}| dy \le C \tag{1.3}$$

The (1.1) *can be extended to a bounded operator in* $L^2(\mathbf{R}^{n+1})$.

Proof For each $f \in C_c^\infty(\mathbf{R}^{n+1})$ by means of (1.2) and (1.3) we have,

$$\int |Kf|^2 dy dx \le (2\pi)^{-n} \sup_{y,\xi} \int |\hat{K}| d\sigma \int |\hat{K}||\hat{f}|^2 d\sigma dy d\xi \le C^2 \|f\|_{L^2}^2$$

An application of the parseval inequality noting the density in $L^2(\mathbf{R}^{n+1})$ of $C_c^\infty(\mathbf{R}^{n+1})$ yields the conclusion of the present lemma.