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ON BASIC SEMICONDUCTOR EQUATIONS WITH HEAT CONDUCTION*

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Abstract We prove the global existence of solution to basic semiconductor equations with heat conduction; If the domain is narrow in one direction, then the basic equations has a unique steady-state which is locally asymptotically stable.
Key Words Super-subsolution; fixed point theorem; L^p estimate and Schauder estimate.
Classification 35K57, 35M10.

1. Introduction

(1) We consider a nonlinear system of partial differential equations arising from semiconductor theory (see [1]):

$$\begin{cases} \Delta\psi = \dfrac{q}{\varepsilon}(n-p-N(x)) & (1.1)\\ \operatorname{div}\vec{J}_n - \dfrac{\partial n}{\partial t} = R(n,p,\theta) & (1.2)\\ \operatorname{div}\vec{J}_p - \dfrac{\partial p}{\partial t} = R(n,p,\theta) & (1.3)\\ k\dfrac{\partial\theta}{\partial t} - \Delta\theta = H(n,p,\psi,\nabla n,\nabla p,\nabla\psi) & (1.4)\end{cases}$$

where ψ is the electrostatic potential, n and p are the densities of mobile holes and electrons respectively, θ is the temperature. $\vec{J}_n = D_n\nabla n - \mu_n n\nabla\psi$, $\vec{J}_p = D_p\nabla p + \mu_p p\nabla\psi$ are the hole and electron current densities, D_n and D_p are the diffusion coefficients for holes and electrons, μ_n and μ_p are the mobility of holes and electrons. R is the net recombination rate. $N(x)$ is the density of ionized impurities. ε and q denote the dielectric permittivity and the unit change. k is associated with the material. We assume $\mu_n, \mu_p, D_n, D_p, \varepsilon, q, k$ are positive constants. $R(n,p,\theta) = r(n,p,\theta)(np - l(\theta))$. H represents local produced heat, one of the simplest forms is $-(\vec{J}_n + \vec{J}_p)\nabla\psi$.

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The system (1.1)–(1.4) governs the transport of mobile carriers in a semiconductor device. For details please see [2] or [3]. Most researchers neglect the influence of change of temperature. Strictly speaking, the heat equation (1.4) should be included.

(2) On boundary condition: In this article we will only consider Dirichlet boundary condition.

(3) Known results and recent developments: there are many results when θ is considered as a constant. On steady-states, the first existence result is established in [4] under condition $R = 0$; for more general existence result of steady-states, we refer to [5–7]. On uniqueness of steady-states, we refer to [8–11] where partial results are included. Generally physical considerations show that one has to expect non-uniqueness of steady-states. On the global existence and uniqueness of solutions of (1.1)–(1.3) (θ is regarded as a constant), see [12], [13], [1]. On asymptotic behavior of solutions of (1.1)–(1.3), partial results are obtained in [14] under very special boundary conditions. On the basic equations with heat conduction, as far as we know, only Seidmann [15] and Seidmann and Troianiello [16] obtained some results. In [15] they proved the existence and uniqueness of solutions of (1.1)–(1.4) and the existence of periodic solutions. In [16] they showed some results on the existence and uniqueness of solutions to (1.1)–(1.4) and the existence of steady-state. No result is known on the asymptotic behavior of the solutions of (1.1)–(1.4).

Recently we learned following interesting results from [17]: if Ω is sufficiently narrow in one direction, θ is a constant, then (1.1)–(1.3) has a unique steady-state and the solutions of (1.1)–(1.3) converge to the unique steady-state exponentially.

(4) Our main results: in this paper we will try to extend the results of [17] to system (1.1)–(1.4): first we established the global existence of solutions of (1.1)–(1.4); if Ω is sufficiently narrow in one direction, then (1.1)–(1.4) has a unique steady-state and it is locally asymptotically stable.

2. Existence and Uniqueness of Solutions of (1.1)–(1.4)

We impose following initial and boundary conditions:

$$\begin{aligned} &n, p, \theta|_{t=0} = n_0(x), p_0(x), \theta_0(x) \\ &n, p, \theta, \psi|_{\partial\Omega} = \bar{n}(x,t), \bar{p}(x,t), \bar{\theta}(x,t), \bar{\psi}(x,t) \end{aligned} \tag{2.1}$$

Theorem 1 *Under following conditions, the system (1.1)–(1.4), (2.1) has a unique solution* $(n, p, \psi, \theta) \in [C^{2+\alpha, 1+\alpha/2}(Q_T)]^4$ *for* $T > 0 : 0 \le r(n, p, \theta) \le r_1$, $0 \le l \le l_1$, $(0,0) \le (\bar{n}, \bar{p})$, $(n_0, p_0) \le (1,1)$, $0 \le N(x) \le \bar{N}$, *where* r, l, H *are Lipschitz continuous,* $r_1, l_1, \bar{N}$ *are positive constants,* $\bar{n}, \bar{p}, \bar{\theta}, \bar{\psi} \in C^{2+\alpha,1+\alpha/2}(Q_T)$, $n_0, p_0, \theta_0 \in C^{2+\alpha}(\bar{\Omega})$, $N(x) \in C^{\alpha}(\bar{\Omega})$, $|H(n, p, \psi, \nabla n, \nabla p, \nabla\psi)| \le H_0(n, p, \psi, \nabla\psi) + H_1(n, p, \psi, \nabla\psi)|\nabla n|^{l_0} + H_2(n, p, \psi, \nabla\psi)|\nabla p|^{l_0}$, *where* H_1, H_2 *are continuous,* l_0 *is some positive constant, and the compatibility conditions on* n_0 *and* $\bar{n}$, p_0 *and* $\bar{p}$, θ_0 *and* $\bar{\theta}$ *are always assumed.*