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Gas-Kinetic BGK Scheme for Three Dimensional Magnetohydrodynamics

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Abstract. The gas-kinetic theory based flux splitting method has been successfully proposed for solving one- and two-dimensional ideal magnetohydrodynamics by Xu et al. [*J. Comput. Phys., 1999; 2000*], respectively. This paper extends the kinetic method to solve three-dimensional ideal magnetohydrodynamics equations, where an adaptive parameter η is used to control the numerical dissipation in the flux splitting method. Several numerical examples are given to demonstrate that the proposed method can achieve high numerical accuracy and resolve strong discontinuous waves in three dimensional ideal MHD problems.

AMS subject classifications: 76M25, 76W05

Key words: The kinetic BGK scheme, magnetohydrodynamics, divergence-free condition.

1. Introduction

The ideal magnetohydrodynamics (MHD) equations are very important in modeling many flow phenomena in astrophysics, space weather, laboratory plasmas, and solar physics etc. Various high-resolution schemes have been developed for the MHD equations in the past two decades. For example, approximate Riemann solvers based on seven or eight waves eigensystems were widely used, see, e.g., [2–5, 8, 11, 16–18, 20, 23, 25, 34]. Tóth and Odstrcil in [27, 28] presented comparisons of some flux corrected transport and total variation diminishing (TVD) schemes as well as various constrained transport methods for the MHD problems. Recently, Han and Tang [13, 14] constructed a divergence-free moving mesh method for two-dimensional ideal MHD system as well as shallow-water MHD

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system based on the reconstruction of the magnetic potential. Due to the non-strictly hyperbolicity of the MHD system, considerable work is required for the validation of the MHD eigensystem. Based on the particle transport mechanism, Croisille et al. and Xu et al. constructed gas-kinetic MHD solvers [10, 26, 31]. Because of the simplicity of the kinetic flux functions, the efficiency becomes one of the advantages in the kinetic approach.

The aim of this paper is to develop a higher-order kinetic BGK scheme for three-dimensional magnetohydrodynamics. The mainly difficulty in multidimensional MHD calculations is to handle the divergence-free constraint for the magnetic field $\vec{B}$, i.e. $\nabla\cdot\vec{B}=0$. Violating this constraint leads to nonphysical plasma transport orthogonal to the magnetic field. Up to now, there are several popular approaches to enforce this condition. The first approach is the projection method of Brackbill and Barnes [7]. In order to impose the divergence free condition for the magnetic field $\vec{B}$, a correction method is enforced in solving the Poisson equation for the scalar potential ϕ, such as $\nabla^2\phi+\nabla\cdot\mathbf{B}=0$, to obtain the corrected magnetic field $\mathbf{B}^c$ through $\mathbf{B}^c=\mathbf{B}+\nabla\phi$, where $\mathbf{B}^c$ becomes a divergence-free field and will be used in the next time step. This technique is commonly used in many MHD solvers [7, 15, 26, 33]. However, in general, the Poisson solver is time consuming on an unstructured mesh or in curvilinear coordinates; and conservation of the total energy may slightly be lost.

The second approach is the eight-wave formulation of the MHD equations suggested by Powell and Aslan [1, 19], who added source terms, which are proportional to the magnetic divergence, to the right hand side of the momentum and total energy equations in the ideal MHD system, respectively. The main disadvantage of this approach is that the 8-wave formulation of the MHD equations becomes non-conservative so that incorrect results may be produced in problems containing strong shocks [27].

The third approach is the constrained transport (CT) method of Evans and Hawley [12], in which a particular finite difference method was constructed on a staggered mesh, maintaining a specific discretization of $\nabla\cdot\vec{B}$. Because of its simplicity, this approach becomes rather popular in recent years, see, e.g., [6, 11, 22]. Tóth [27] introduced a finite-volume interpretation of the CT schemes that place all of the variables at the cell center. However, the idea seems to be difficult to apply to an adaptive mesh (refinement mesh or moving mesh). It is worth noting that most of the existing CT methods are designed on a rectangle or cubic mesh. Another way to keep the magnetic field divergence-free is to directly solve the magnetic potential equations instead of the induction equation in the ideal MHD system, see [9, 12, 21]. The disadvantage of this approach is that the order of spatial derivatives increases by one, which reduces the order of accuracy by one.

The paper is organized as follows. Section 2 introduces the governing equations for the three-dimensional ideal MHDs. Section 3 develops a higher-order kinetic BGK scheme for three-dimensional magnetohydrodynamics. The adjust parameter η is adaptively defined in the BGK scheme. We correct the magnetic field of the base MHD solver by the projection method. Numerical experiments are carried out in Sections 4 on two benchmark examples, which are the spherical explosion problem and the spherical cloud and shock wave interaction problem. Finally, we conclude this work by giving a few remarks in Section 5.