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Asymptotic Formulas for Thermography Based Recovery of Anomalies

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Abstract. We start from a realistic half space model for thermal imaging, which we then use to develop a mathematical asymptotic analysis well suited for the design of reconstruction algorithms. We seek to reconstruct thermal anomalies only through their rough features. With this way our proposed algorithms are stable against measurement noise and geometry perturbations. Based on rigorous asymptotic estimates, we first obtain an approximation for the temperature profile which we then use to design noniterative detection algorithms. We show on numerical simulations evidence that they are accurate and robust. Moreover, we provide a mathematical model for ultrasonic temperature imaging, which is an important technique in cancerous tissue ablation therapy.

AMS subject classifications: 35R20, 35B30

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1. Introduction

Medical thermal imaging has become a procedure of choice in the screening for breast, skin, or liver cancer [10]. It has the ability to identify various stages of disease development, and can pick up early stages which usually elude traditional anatomical examinations. Thermal imaging relies on the fact that chemical and blood vessel activity in pre-cancerous tissue and its surroundings are higher than in healthy tissue. Pre-cancerous and cancerous areas are characterized by heightened metabolism and require an abundant stream of nutrients to maintain growth. These extra nutrients are transported through various channels such as increased chemical activity, enhanced blood stream, and creation

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of new blood vessels (neoangiogenesis) [15]. This process results in a local increase in temperature.

Detection of these small temperature variations is made possible by state of the art imaging techniques. They involve ultra-sensitive thermal cameras and sophisticated software in detecting, analyzing, and producing high-resolution thermal images of vascular changes. More precisely, medical thermal imaging technique proceeds as follows: an infrared scanning device is used to convert infrared radiation emitted from the skin surface to electrical impulses. Those are then plotted on a color monitor. This map of body surface temperature is referred to as a thermogram. The spectrum of colors corresponds to a scale of infrared radiation emitted from the body surface. Since temperature distribution is highly isotropic in healthy tissue, subtle temperature anisotropies produce a clear imprint. See [1,13].

Thermal imaging is a very reliable technology. In fact, clinical studies have shown that thermal imaging has an average sensitivity and specificity of 90% when applied to screening of breast tissue. As of today, an abnormal infrared image is the single most important marker of high risk of onset breast cancer. Thermal imaging may also be used for different purposes such as

(i) assessing the extent of a previously diagnosed lesion;

(ii) localizing an abnormal area not previously identified, so further diagnostic tests can be performed;

(iii) detecting early lesions before they are clinically apparent;

(iv) guiding thermal ablation therapies.

In this paper, following an asymptotic formalism in much the same spirit as the recent texts [2, 5], we perform a quantitative study of temperature perturbation due to small thermal anomalies and design algorithms for localizing these anomalies and estimating their size. We start from a realistic model in half space with convective boundary condition on the surface.

Since the exterior temperature is imposed, a convective boundary condition must be used for the model to be physically relevant. Moreover, since the anomalies are expected to be at some distance away from the boundary, the resulting change in surface temperature will be very small: a realistic reconstruction method must take into account inevitable noise blurring of measurements as well as the ill posed nature of the inverse problem. Additionally, we believe that we can realistically assume that the temperature is a known constant far away from the surface: this models temperature far inside the human body, assumed to be constant. In essence, measurements will be made on a planar surface, assumed to be large enough compared to the size of anomalies. In this case a half space formulation provides a viable model. We refer the reader to [7], where such half space formulations are scrutinized.

It is noteworthy that our results can be applied to other types of thermography problems, such as the detection of buried objects in the underground. We seek to reconstruct