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Hybrid Surface Mesh Adaptation for Climate Modeling

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Abstract. Solution-driven mesh adaptation is becoming quite popular for spatial error control in the numerical simulation of complex computational physics applications, such as climate modeling. Typically, spatial adaptation is achieved by element subdivision (h adaptation) with a primary goal of resolving the local length scales of interest. A second, less-popular method of spatial adaptivity is called "mesh motion" (r adaptation); the smooth repositioning of mesh node points aimed at resizing existing elements to capture the local length scales. This paper proposes an adaptation method based on a combination of both element subdivision and node point repositioning (rh adaptation). By combining these two methods using the notion of a *mobility function*, the proposed approach seeks to increase the flexibility and extensibility of mesh motion algorithms while providing a somewhat smoother transition between refined regions than is produced by element subdivision alone. Further, in an attempt to support the requirements of a very general class of climate simulation applications, the proposed method is designed to accommodate unstructured, polygonal mesh topologies in addition to the most popular mesh types.

AMS subject classifications: 52B10, 65D18, 68U05, 68U07

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1. Introduction

Mesh generation is an important consideration in the area of numerical simulation of physical phenomena. Indeed, the accuracy and convergence of approaches using mesh-based numerical methods are strongly dependent on the intrinsic characteristics of the mesh being used. The "quality" of a mesh is loosely termed as the degree in which a particular mesh supports a given simulation. For transient calculations, the mesh supporting the calculation must not only be of high quality initially, it also must effectively support the requirements of the dynamic simulation as it evolves. Generally, solution features will

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develop and move across the domain; these features must be resolved as they propogate through the mesh.

One approach to mesh adaptation is to begin with an appropriate initial mesh, and sweep over this mesh in both space and time calculating an *error metric*. This error metric is chosen to represent some quantification of the error in the solution in each mesh element. As the sweep progresses, provided that this metric is above a certain value, the mesh element is subdivided to reduce its area (or volume) and hence the value of the error metric inside the refined portion of the original element.

This method of mesh element subdivision is commonly called *h* refinement, or *adaptive mesh refinement (AMR)*. An example of *h* refinement on an orographic map of the Himalayas mountains is shown in Fig. 1. In this example, triangular, quadrilateral, and polygonal meshes are adapted to better capture the Earth's orography field (the local average surface elevation). The upper-left inset of the figure shows a color intensity map of the orographic value of the Earth's surface, where blue indicates sea level and red indicates the highest elevation regions. The upper-right inset shows a quadrilateral mesh on the Earth's surface that is *h* refined using the intensity of the orographic value as a refinement metric. The lower-left inset shows an *h* refined triangle surface mesh, where the lower-right inset shows a refined polygonal mesh.

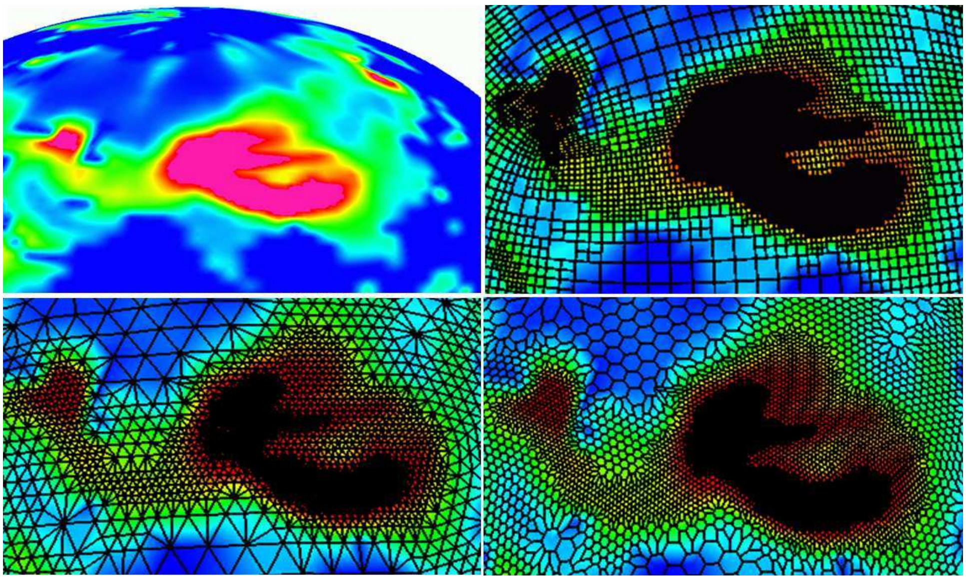

Figure 1: An example of *h*-refinement driven by an orographic scalar on a mesh of the Earth's surface. Upper-left figure shows elevation; blue is near sea-level and red indicates high elevation regions. The upper-right figure shows a refined quadrilateral mesh while the lower-left and lower-right figures show triangular and polyhedral meshes, respectively.

While the *h* refinement algorithm is a general approach that is effective on all element shapes and for all length scales, the method results in abrupt (*i.e.,* non-smooth) variations in element area or volume. These non-smooth transitions may impact local simulation accuracy at and near the transition elements. This limitation of element subdivision has to