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PARALLEL DATA PARTITIONING STRATEGY IN SOLVING LARGE SCALE ELECTROMAGNETIC SCATTERING PROBLEMS

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Abstract. The multilevel fast multipole algorithm (MLFMA) has shown great efficiency in solving large scale electromagnetic scattering problems. However, when unknowns become up to tens of millions, it is not trivial to keep high performance because of the complicated structure and calculation of MLFMA. In order to get rid of the bottleneck caused by load balancing, a parallel data partitioning strategy is proposed based on the hierarchical structure of an oct-tree of MLFMA. We present our data partitioning strategy in the light of different layers' properties including the processing of three kinds of layers in the tree and a fine-grained decomposition. We also put forward a solution of a coexisting data correlating problem, using a transition layer. Meanwhile, with the purpose of minimizing communication time in distributed memory system, a redundant technique is applied in the distributed layer. Parallel efficiency analysis demonstrates that the computational cost in parallelization of MLFMA can be asymptotically cut, and a high parallel efficiency can be obtained in our implementation.

Key words. Multilevel Fast Multipole Algorithm (MLFMA), Parallel Data Partitioning Strategy, Hierarchical Structure, Data Correlating Problem, and Redundant Technique.

1. Introduction

To achieve the fast computing characteristic of large scale electromagnetic problems, the Multilevel Fast Multipole Method (MLFMA) is applied, as well as Message Passing Interface (MPI) for network communications among processors. MLFMA was optimized by Song and Chew [1] in 1995, which has been widely used in recent years. Song and Chew implemented the MLFMA with $O(NlogN)$ complexity, where N is the number of unknowns, and the memory requirement using translation, interpolation, anterpolation (adjoint interpolation), and a grid-tree data structure.

For the actual demand, we hope to develop a program to solute a full-sized aircraft problem that could run concurrently from single workstations to network-linked clusters. For the sake of a full-sized airplane, unknowns could be up to tens of millions. Although MLFMA has shown its high performance in reducing the computational complexity and the memory complexity of Matrix Vector Multiplications (MVMs) from $O(N^2)$ to $O(NlogN)$, when N extends to millions, several encumbrances have to be faced. And simple parallelization strategies usually fail to provide efficient solutions, owing to massive communications, poor load-balancing and necessary duplications. Advanced parallelization techniques have been proposed to improve the parallelization of MLFMA by using preconditioning strategies [2], extensively investigate the parallelization of MLFMA, identify the bottlenecks and provide remedial procedures [3], and even a novel method called nondirective stable plane wave multilevel fast multipole algorithm is developed to evaluate the low-frequency interactions which cannot be managed by MLFMA [4]. Especially

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after answered the question that whether 10 million is big [5], Velamparambil and Chew analyzed the communication pattern, computational behavior and studied the scalability of a distributed memory implementation of MLFMA called ScaleME [6].

Recently, we developed a hierarchical partitioning strategy to fit for the multilevel structure of MLFMA. With this method an enhanced load-balancing is obtained, parallelization of MLFMA is improved significantly, and it has become possible for us to solve a three-dimensional full aircraft discretized up to 10 million unknowns with optimal parallel efficiency. In this paper, we present the details of a parallel MLFMA data partitioning implementation including investigating the parallelization procedure by focusing on different parts of an oct-tree and identifying a fine-grained data decomposition. Our approach involves the partitioning strategies to distribute tasks equally among processors and minimize the interprocessor communications.

The rest of the paper is organized as follows. In section 2, the MLFMA equations we use are briefly described, as well as a data collecting scheme and the layout of the oct-tree of MLFMA in our implementation. Section 3 narrates a fine-grained data decomposition, followed by the detail partitioning strategies of each layer of the oct-tree in section 4. The results are analyzed in section 5, and section 6 introduces our conclusion and future work.

2. Background

2.1. Multilevel Fast Multipole Algorithm (MLFMA). For the solution of the electromagnetic scattering problems involving three-dimensional conducting bodies with arbitrary shapes, Multilevel Fast Multipole Algorithm, which is detailed in [7]-[10], performs efficiently together with the Fast Multipole Method (FMM) [11] and a large problem can be solved iteratively, where the required Matrix-Vector Multiplications (MVMs) are involved. The application of boundary conditions for the electric filed and the magnetic field on the surface of an object leads to the Electric Field Integral Equation (EFIE) and the Magnetic Field Integral Equation (MFIE), respectively. For closed surfaces, EFIE and MFIE can be combined to obtain the Combined Field Integral Equation (CFIE). These three equations are briefly described as follows and considered as the point of departure in our work.

- EFIE

$$\widehat{n} \times L(\overrightarrow{J}) = \widehat{n} \times \overrightarrow{E}'. \tag{1}$$

where

$$L(\overrightarrow{J}) = -\overrightarrow{E}^s = jk\eta \int_s [\overrightarrow{J}((\overrightarrow{r})')G + \frac{1}{k^2} \bigtriangledown' \cdot \overrightarrow{J}((\overrightarrow{r})') \bigtriangledown G] ds'.$$

- MFIE

$$\frac{1}{2} J(r) + \overrightarrow{n} \times K(\overrightarrow{J}) = \overrightarrow{n} \times \overrightarrow{H}^i. \tag{2}$$

where

$$K(\overrightarrow{J}) = \int_s \overrightarrow{J}(\overrightarrow{r}) \times \bigtriangledown G ds'.$$

- CFIE