Commun. Comput. Phys.
doi: 10.4208/cicp.220513.070514a

Vol. **16**, No. 5, pp. 1181-1200
November 2014

A New Approach for Error Reduction in the Volume Penalization Method

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Received 22 May 2013; Accepted (in revised version) 7 May 2014

Communicated by Jie Shen

Available online 29 August 2014

Abstract. A new approach for reducing error of the volume penalization method is proposed. The mask function is modified by shifting the interface between solid and fluid by $\sqrt{\nu\eta}$ toward the fluid region, where ν and η are the viscosity and the permeability, respectively. The shift length $\sqrt{\nu\eta}$ is derived from the analytical solution of the one-dimensional diffusion equation with a penalization term. The effect of the error reduction is verified numerically for the one-dimensional diffusion equation, Burgers' equation, and the two-dimensional Navier-Stokes equations. The results show that the numerical error is reduced except in the vicinity of the interface showing overall second-order accuracy, while it converges to a non-zero constant value as the number of grid points increases for the original mask function. However, the new approach is effective when the grid resolution is sufficiently high so that the boundary layer, whose width is proportional to $\sqrt{\nu\eta}$, is resolved. Hence, the approach should be used when an appropriate combination of ν and η is chosen with a given numerical grid.

AMS subject classifications: 60-08

Key words: Volume penalization method, immersed boundary method, compact scheme, error reduction.

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1 Introduction

Flows around solid bodies have been investigated in a wide variety of fields in science and engineering. Computational fluid dynamics has advantages in both visualizing flow fields and providing detailed data over experiments. The flows around solid objects are often calculated using either a body-fitted grid system to impose boundary conditions or a set of appropriate orthogonal functions which satisfy the boundary conditions to expand the flow variables. However, if there exist complex-shaped solid bodies or bodies which move or deform in the flow, it is not easy to generate a body-fitted grid system or to find a set of orthogonal functions; efficient computation is not possible at low cost by these methods. The volume penalization (VP) method is one of the alternative methods to simulate flows in these complicated situations.

The VP method is one of the immersed boundary methods which are classified into two types: *the continuous forcing approach* in which an external force term is added to a continuous equation of motion and *the discrete forcing approach* in which the force term is added to a discretized one [13]. The VP method is the former type. One can use it with the Fourier pseudo-spectral method; many flows in which multiple solid bodies exist [9,10,14,15], the flows inside rigid boundaries [16,17], and the flows around moving bodies [11] have been simulated by the VP method. Moreover, the VP method can be used with Chebyshev pseudo-spectral method, wavelet solvers, and other high-precision methods [8].

In the VP method, a solid body is regarded as porous medium of low permeability. There are two types of penalization modeling. One is the L^2 penalization: the Navier-Stokes (N-S) equation is converted to the Darcy equation in the solid body; and the other is the H^1 penalization: the N-S equation is transformed to the Brinkman equation in the solid body [1, 2]. In the L^2 penalization, a damping force term which is called a penalization term and has a mask function χ and the permeability η is added to the equation of motion. Usually the step function, which is 0 in the fluid region and 1 in the solid region, is chosen as χ. The mask function activates the penalization term in the solid region so that the penalized N-S equation turns into the Darcy equation.

One of the advantages of the VP method is that there are rigorous results about convergence. As permeability tends to zero, the penalized solution converges to the solution of the original (non penalized) problem with Dirichlet-type boundary conditions, e.g. no-slip boundary conditions. Angot *et al.* proved mathematically that the upper bound for the difference between the solutions of the original and penalized N-S equations, is $\mathcal{O}(\eta^{1/4})$ in the fluid region [1]. This upper bound is improved to $\mathcal{O}(\eta^{1/2})$ by Carbou and Fabrie [4]. Kevlahan and Ghidaglia [9] considered a stokes flow over a flat plate whose dynamics is reduced to the one-dimensional diffusion equation and showed analytically that the error between the original and penalized solutions is $\mathcal{O}(\eta^{1/2})$ in the fluid region. Recently, Kadoch *et al.* applied the VP method to problems with Neumann-type boundary conditions, e.g. no-flux conditions [7]. They draw the same conclusion as Carbou and Fabrie [4] in the convergence property.