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# Numerical Boundary Conditions for Specular Reflection in a Level-Sets-Based Wavefront Propagation Method

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**Abstract.** We study the simulation of specular reflection in a level set method implementation for wavefront propagation in high frequency acoustics using WENO spatial operators. To implement WENO efficiently and maintain convergence rate, a rectangular grid is used over the physical space. When the physical domain does not conform to the rectangular grid, appropriate boundary conditions to represent reflection must be derived to apply at grid locations that are not coincident with the reflecting boundary. A related problem is the extraction of the normal vectors to the boundary, which are required to formulate the reflection condition. A separate level set method is applied to pre-compute the boundary normals which are then stored for use in the wavefront method. Two approaches to handling the reflection boundary condition are proposed and studied: one uses an approximation to the boundary location, and the other uses a local reflection principle. The second method is shown to produce superior results.

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## 1 Introduction

### 1.1 Motivation

High frequency wave propagation models are of vital importance to understanding the propagation of sound in shallow water environments. Applications such as acoustic tomography [1] and underwater communications [2] rely on ray tracing for system design

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and performance prediction. The frequencies useful for such applications in shallow water are at least on the order of 1kHz and can range into the MHz regime, rendering full wave models impractical in simulations. On the other hand, ray tracing has many known limitations. In particular, the Lagrangian nature of the ray trace model leads to difficulty resolving wave arrivals at a fixed location in space. This is especially true in range-dependent, shallow water environments. The goal of this work is to devise stable, accurate, numerical boundary conditions for reflection in a level set method implementation that solves the high frequency equation for the acoustic phase, thus tracing entire wavefronts on a fixed grid in space. A practical wavefront model for shallow water acoustics would allow for improved accuracy in such harsh environments, where multipath propagation is evident due to multiple reflections off of the sea surface and bottom. Such a model would also be more robust to perturbations in material properties. For example, Godin [3] studied the effect of small perturbations in the sound speed on rays, showing that the resulting displacement of rays *across* the wavefront are much smaller than displacements *along* the wavefront. This type of relative stability of wavefronts is critical to practical simulations due to the uncertainty present in sound speed profiles derived from at-sea data collection.

This study is based on a method laid out in [4], introducing an implementation of the level set method intended for a two-dimensional shallow water acoustics application. The algorithm is based on the level set method for geometric optics [5]. The level set equations are solved using a fifth order Weighted Essentially Non-Oscillatory (WENO) method for the spatial operator [6] combined with third order Total Variation Diminishing Runge-Kutta (TVDRK) time integration [7] for stability. This choice is motivated by the presence of a discontinuity in the phase space at a reflecting boundary [4]. While the discontinuity is a consequence of the rectangular scatterers presented in that work, this is not the case in general, but even in the benign case of a flat reflecting boundary, a sharp cusp forms at the boundary in the reduced phase space. The examples presented in [4] are limited by the types of domain geometry that could be handled under the reflection boundary condition to either grid-conforming rectangular domains, or a domain with a linear boundary. For the linear boundary case, the boundary did not conform to the grid and an approximation was applied in order to provide the required boundary conditions for the numerical solver. This approach is discussed in greater detail in Section 2.3.1. The goal of the present work is to extend the method to accommodate more realistic domain geometry, and improve the accuracy of the reflection boundary condition. To achieve this, one could modify the rectangular grid to conform to the geometry, as in [8], however in the general case, this affects the convergence of WENO. Refining the grid to match the geometry can result in strong restrictions on the CFL condition. Another approach would be to use finite elements or finite volume methods on an unstructured grid, but this complicates the implementation of WENO, especially in high dimensions (for a dimension $d$ physical space, the reduced phase space has dimension $2d-1$). Alternately, the approach taken in this work is to embed the reflector in the uniform, rectangular grid and derive appropriate boundary conditions to apply at grid points adjacent to the reflector.