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Picard-Newton Iterative Method with Time Step Control for Multimaterial Non-Equilibrium Radiation Diffusion Problem

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Abstract. For a new nonlinear iterative method named as Picard-Newton (P-N) iterative method for the solution of the time-dependent reaction-diffusion systems, which arise in non-equilibrium radiation diffusion applications, two time step control methods are investigated and a study of temporal accuracy of a first order time integration is presented. The non-equilibrium radiation diffusion problems with flux limiter are considered, which appends pesky complexity and nonlinearity to the diffusion coefficient. Numerical results are presented to demonstrate that compared with Picard method, for a desired accuracy, significant increase in solution efficiency can be obtained by Picard-Newton method with the suitable time step size selection.

AMS subject classifications: 65M06, 65M08, 65M12

Key words: Non-equilibrium radiation diffusion, Picard, Picard-Newton based on linearization-discretization, nonlinear iterative method, time step control.

1 Introduction

Picard nonlinear iterative method is a globally convergent method and has been used extensively in many applications. However its iterative convergence rate is only one order, and its solution efficiency is low in solving some practical problems. So Newton-like methods with super-linear convergence must be introduced, and these nonlinear Newton-Krylov solution techniques [2–11] have been developed to ensure nonlinearities convergence. It is well-known (see, e.g., [5–8]) that there are two main obstacles which have prevented people from using Newton-like methods for large scale multi-physics simulations.

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The first obstacle in Newton-like methods for solving some practical multi-physical problems is the need for evaluating the entries of the Jacobian matrix. To overcome this obstacle we have proposed in [20, 21] a new nonlinear iterative method named as Picard-Newton (P-N) iterative method for solving numerically nonlinear diffusion equations and multimaterial non-equilibrium radiation diffusion problems. To construct a new nonlinear iteration method, we directly design iterative method for the time discretized nonlinear PDEs instead of nonlinear algebraic system from implicit discretization scheme. By linearizing the time discretized nonlinear PDEs first and discretizing the resulting linear PDEs next, which is named as the linearization-discretization (LD) approach, we have devised the Picard-Newton iterative scheme which gives a specific procedure to the formation of (exact or approximated) Jacobian matrix. It follows that the first obstacle can be overcome through the LD approach.

The second obstacle is that Newton-like methods cannot be expected always to converge since they are locally convergent. In other words, an initial guess inside the radius of convergence is required for Newton-like methods to converge, while for time-dependent or transient problems the initial guess is usually the converged solution from the previous time step. A reliable time step selection is necessary for nonlinear iterative methods to ensure the methods to operate with desired accuracy. Then time step control is essentially important for fast and accurate numerical solution, especially when Newton-like method is employed.

In addition to the difficulties of evaluating the Jacobian matrix and the radius of convergence being small, on each iteration step of Newton-like method for nonlinear systems the cost of solving the linear systems to arrive at the new iteration values might be very expensive, which depends mainly on the matrix properties of the linear systems. In other words, since the matrix properties of the linear systems corresponding to Newton method are usually worse than those corresponding to Picard method, the computational cost of the original Newton method at each nonlinear iteration step is more expensive than that of Picard method.

We hope our P-N method can accelerate the existing Picard method. The P-N method can be formulated by adding certain convective terms of one order to the Picard method. When the one order terms are discretized by centered scheme, the resulting nonlinear iteration is named as P-NC iterative scheme, while discretized by upwind scheme, the resulting nonlinear iteration is named as P-NU iterative scheme. It is worth to point out that the P-NC may be equivalent to standard Newton method, while the P-NU is new and is an improvement over Picard and Newton methods. From the construction of our P-NU method we can see that the diagonal dominance of the matrix from P-NU is not worse than that from Picard method, and then it is possible to reduce remarkably computational time for each step of nonlinear iteration.

Since our P-N has overcome the first obstacle through the LD approach, it remains to demonstrate our P-N need not introduce more strict time steps than that used for Picard. In this paper, two time step control methods are investigated and a study of temporal accuracy of a first order time integration is presented for P-N iteration. We are