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# Finite Volume Methods for Wave Propagation in Stratified Magneto-Atmospheres

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**Abstract.** We present a model for simulating wave propagation in stratified magneto-atmospheres. The model is based on equations of ideal MHD together with gravitational source terms. In addition, we present suitable boundary data and steady states to model wave propagation. A finite volume framework is developed to simulate the waves. The framework is based on HLL and Roe type approximate Riemann solvers for numerical fluxes, a positivity preserving fractional steps method for discretizing the source and modified characteristic and Neumann type numerical boundary conditions. Second-order spatial and temporal accuracy is obtained by using an ENO piecewise linear reconstruction and a stability preserving Runge-Kutta method respectively. The boundary closures are suitably modified to ensure mass balance. The numerical framework is tested on a variety of test problems both for hydrodynamic as well as magnetohydrodynamic configurations. It is observed that only suitable choices of HLL solvers for the numerical fluxes and balanced Neumann type boundary closures yield stable results for numerical wave propagation in the presence of complex magnetic fields.

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# 1 Introduction

There is considerable interest in the astrophysics community regarding the problem of wave propagation in magnetized stellar atmospheres. The main theme of this research is to determine how convection generated waves transport and deposit energy in the overlying chromospheric and coronal plasmas. Models aim to explain the observed energy distribution in interesting astrophysical objects like the sun. The mathematical description of the underlying physical processes in realistic magneto-atmospheres is extremely complicated. The models for this wave heating problem include the equations of ideal magneto-hydrodynamics (MHD) together with complicated source and diffusion terms. In addition, radiative transfer and non-equilibrium thermodynamics also play leading roles. These models are described by systems of nonlinear partial differential and integral equations in three dimensions, coupled with realistic initial and boundary data. The issue of determining model parameters along with proper initial and boundary data requires considerable observational work.

It is not possible to obtain analytical solutions for the full model or even extremely simplified versions of it. Also for these equations, theoretical results concerning existence, uniqueness and qualitative behavior are currently unavailable. Therefore, in order to investigate these models, one must resort to numerical methods. Even this task faces formidable difficulties due to nonlinearity and sheer computational complexity. A detailed account of the physical processes involved in wave propagation along with extensive references to the corresponding astrophysics literature can be found in [1, 2].

In [1,2], the authors consider a relatively simple model for wave propagation in the solar atmosphere. This model takes into account the equations of compressible ideal MHD along with gravitational source terms, supplemented by a description of the underlying steady states. Waves in the "solar" atmosphere are modeled by inducing perturbations of these steady states. We adopt the modeling framework of the above papers as a starting point of this work and develop a class of schemes of the finite volume type to simulate this model. A complete description of these schemes involves suitable approximate Riemann solvers for the ideal MHD equations, proper treatment of the gravitational source term and an appropriate implementation of boundary conditions.

The core of the model we consider consists of the equations of ideal MHD. Consequently, most of the computational effort is directed at MHD solvers. The MHD equations are an example of a system of non-linear hyperbolic conservation laws. Solutions of these equations develop discontinuities such as shock waves and contact discontinuities even for smooth initial data. Furthermore, the MHD equations are not strictly hyperbolic and contain a large number of waves. Some of the characteristic fields are not convex (i.e. genuinely nonlinear except in some subset of state space), and the resulting solutions can have intermediate and compound shocks. All these issues have to be addressed in order to design efficient numerical methods for ideal MHD.

Finite volume methods are a popular type of numerical framework for approximating solutions to conservation laws. These methods are based on approximating the integral