INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING
Volume 6, Number 3, Pages 375–388

WELL FLOW MODELS FOR VARIOUS NUMERICAL METHODS

ZHANGXIN CHEN AND YOUQIAN ZHANG

This paper is dedicated to the special occasion of Professor Roland Glowinski's 70th birthdate.

Abstract. Numerical simulation of fluid flow and transport processes in the subsurface must account for the presence of wells. The pressure at a gridblock that contains a well is different from the average pressure in that block and different from the flowing bottom hole pressure for the well [17]. Various finite difference well models have been developed to account for the difference. This paper presents a systematical derivation of well models for other numerical methods such as standard finite element, control volume finite element, and mixed finite element methods. Numerical results for a simple well example illustrating local grid refinement effects are given to validate these well models. The well models have particular applications to groundwater hydrology and petroleum reservoirs.

Key words. Well models, petroleum reservoirs, aquifer remediation, finite difference, finite element, control volume finite element, mixed finite element, fluid flow, numerical experiments

1. Introduction

Numerical simulation of fluid flow and transport processes in the subsurface must account for the presence of wells. The pressure at a gridblock that contains a well is different from the average pressure in that block and different from the flowing bottom hole pressure for the well [17]. The difficulty in modeling wells in a field scale numerical simulation is that the region where pressure gradients are the largest is closest to a well and is far smaller than the spatial size of gridblocks. Using local grid refinement around the well can alleviate this problem but can lead to an impratical restriction on time step sizes in the numerical simulation [5]. The fundamental task in modeling wells is to model flows into the wellbore accurately and to develop accurate well equations that allow the computation of the bottom hole pressure when a production or injection rate is given, or the computation of the rate when this pressure is known.

The first theoretical study of well equations was given by Peaceman [17] for cell-centered finite difference methods on square grids for single phase flow. Peaceman's study gave a proper interpretation of a well-block pressure, and indicated how it relates to the flowing bottom hole pressure. The importance of his study is that the computed block pressure is associated with the steady-sate pressure for the actual well at an equivalent radius r_e. For a square grid with a grid size h, Peaceman derived a formula for r_e by three different approaches: (1) analytically by assuming that the pressure in the blocks adjacent to the well block is computed exactly by the radial flow model, obtaining $r_e = 0.208h$, (2) numerically by solving the pressure equation on a sequence of grids, deriving $r_e = 0.2h$, and (3) by solving exactly the system of difference equations and using the equation for the pressure drop between the injector and producer in a repeated five-spot pattern problem, finding $r_e = 0.1987h$. From these approaches, he concluded that $r_e \approx 0.2h$.

Received by the editors August 30, 2008.
2000 *Mathematics Subject Classification.* 65N30, 65N10, 76S05, 76T05.

Peaceman's finite difference well models on square grids have been extended in various directions, including to rectangular grids, anisotropic reservoirs, horizontal wells, and multiphase flows and to incorporating gravity force, skin, and non-Darcy effects. Peaceman himself extended his classical well model [17] to more general scenarios [18] where rectangular grids and anisotropic permeabilities are allowed. For the treatment of arbitrary well locations and horizontal wells, the reader can refer to [2, 19]. Lee and Milliken [13] studied an arbitrary monobore well in a layered system of laterally infinite extent. They combined a semianalytical solution based on slender body theory with a finite difference pressure solution with lateral pressure boundary conditions described by the semianalytical solution. Ding [8] introduced a layer potential function to obtain a steady state pressure distribution in the vicinity of the well. Furthermore, he adjusted well block transmissibilities to account for radial flow. Later, Ding and Jeannin [9] developed a multipoint discretization in a curvilinear coordinate system and used the discretization coefficient of an elliptic equation as the well index. Recently, Wolfsteiner et al. [22] extended Peaceman's well models to account for different well configurations in heterogeneous porous media. More recently, Chen and Yue [6] derived a well model by introducing multiscale basis functions that resolve well singularity, and Aarnes [1] proposed a modified mixed multiscale finite element method that can account for radial flow near a well. Finally, Ewing et al. [10] and Garanzha et al. [11] developed numerical well models that account for non-Darcy effects.

As far as the authors know, however, most of these existing well models have been developed for finite difference methods [8, 17, 19]. On the other hand, finite element methods have been successfully applied for numerical simulation of fluid flow and transport processes in the subsurface due to their intrinsic grid flexibility [5]. Thus it is clear that, to use finite element approximations in the presence of wells, accurate well models must be derived for this important class of numerical methods.

This paper presents a systematical derivation of well models for finite element approximations of three types: (1) standard finite element methods, (2) control volume finite element methods, and (3) mixed finite element methods. Extensions of these numerical well models to anisotropic reservoirs, horizontal wells, and multiphase flows and to incorporating gravity forces and skin factors are also discussed. For uniform grids and isotropic reservoirs, Peaceman's second approach for deriving well models will be used. When the grids are nonuniform or the reservoirs are anisotropic, we will make remarks on Peaceman's third approach. Numerical results for a simple well example illustrating local grid refinement effects are given to validate the well models derived. To motivate the derivation of finite element well models, the derivation of finite difference models is briefly reviewed.

The rest of the paper is organized as follows. The development of well equations requires the use of analytical formulas, which is given in the second section. In the third section, finite difference models are reviewed. The derivation of well models for the standard, control volume, and mixed finite element methods is carried out, respectively, in the fourth, fifth, and sixth sections. The seventh section is devoted to numerical results. The well model equations derived in this paper have particular applications to numerical simulation of aquifer remediation and of enhanced oil recovery, for example.