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A New Stable Version of the SPH Method in Lagrangian Coordinates

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Abstract. The purpose of this paper is to solve some of the trouble spots of the classical SPH method by proposing an alternative approach. First, we focus on the problem of the stability for two different SPH schemes, one is based on the approach of Vila [25] and another is proposed in this article which mimics the classical 1D Lax-Wendroff scheme. In both approaches the classical SPH artificial viscosity term is removed preserving nevertheless the linear stability of the methods, demonstrated via the von Neumann stability analysis. Moreover, the issue of the consistency for the equations of gas dynamics is analyzed. An alternative approach is proposed that consists of using Godunov-type SPH schemes in Lagrangian coordinates. This not only provides an improvement in accuracy of the numerical solutions, but also assures that the consistency condition on the gradient of the kernel function is satisfied using an equidistant distribution of particles in Lagrangian mass coordinates. Three different Riemann solvers are implemented for the first-order Godunov type SPH schemes in Lagrangian coordinates, namely the Godunov flux based on the exact Riemann solver, the Rusanov flux and a new modified Roe flux, following the work of Munz [17]. Some well-known numerical 1D shock tube test cases [22] are solved, comparing the numerical solutions of the Godunov-type SPH schemes in Lagrangian coordinates with the first-order Godunov finite volume method in Eulerian coordinates and the standard SPH scheme with Monaghan's viscosity term.

AMS subject classifications: 52B10, 65D18, 68U05, 68U07

Key words: SPH, meshfree particle methods, Riemann solvers, gas dynamics in Lagrangian coordinates, Godunov type schemes.

1 Introduction

The Smoothed Particle Hydrodynamics (SPH) method was originally introduced by Lucy [11], Gingold and Monaghan [5]. It is one of the earliest particle methods in compu-

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tational mechanics and it was devised to simulate a wide variety of problems in astrophysics. Like many meshfree methods, the SPH scheme is based on the Lagrangian approach and it is able to handle problems characterized by large deformations, moving discontinuities and critical mesh distortions.

In the SPH scheme, the generic continuum, such as a fluid, is discretized by a finite set of discrete values defined at observation points, the so-called particles. Each point is not fixed on a mesh, but it moves with the velocity of the fluid and the interactions of each other are determined by a local function, the smoothing kernel. This function is the essential feature of the SPH scheme and it assigns the weights of each particles based on the reciprocal positions of the interpolating points.

Different smoothing functions have been used in the SPH method as seen in the literature [9]. The most widely used kernel functions in the SPH simulations are the Gaussian and the cubic $B-$spline of Monaghan and Lattanzio [15]. In spite of the interesting mathematical properties of the Gaussian function, most practical work relies on the monotone splines [1]. In fact, using the splines and consequently a smaller support, one can obtain more accurate numerical solutions and more efficiency, from the computational point of view. Unfortunately, even the choice of a spline function can not assure us that the consistency conditions on the kernel are always satisfied [9], because the accuracy of the numerical solution depends also on the distribution of the observation points inside the compact support. This effect is emphasized near the boundaries, when the kernel support leaves the numerical domain and thus the distribution of the particles is unbalanced, but it can be also significant within the computational domain, when the particles are placed irregularly. To solve this issue, a new approach is proposed, which is based on an uniform distribution of the particles in the Lagrangian mass coordinates.

Moreover, the classical SPH method suffers from several well-known numerical problems, such as particle interpenetration in high Mach number flows [13] or the so-called tensile instability [14]. Generally one deals with these issues using various artificial pressure and viscosity terms as introduced by Monaghan [12] in the motion and thermal energy equations, but it does not solve all the issues. In fact, a von Neumann analysis was carried out by Balsara [1] on the SPH method with Monaghan's artificial viscosity term. Unfortunately, only a small range of ratios of smoothing length to particle distance for a specified choice of kernel function leads to stable continuum behavior. Based on that finding we deduce that none of the currently used SPH kernels represents a particularly good choice using Monaghan's viscosity term [1]. In spite of that, up to this day the viscosity term proposed by Monaghan has been mostly used.

An alternative approach has been recently proposed by Vila [25] and by Moussa and Vila [16], who studied the convergence of SPH using approximate Riemann solvers instead of the artificial viscosity. Moreover, Parshikov *et al.* [18], according to Godunov schemes in the Finite Volume method, use the result of the Riemann problem in the calculation of the numerical flux. Good results have been also obtained by Cha and Whitworth [3], who have applied the Riemann solver of van Leer [23, 24] to isothermal hydrodynamics. A recent improvement of the order of accuracy of SPH comes from Inut-