COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. **2**, No. 2, pp. 310-333

Commun. Comput. Phys.
April 2007

# High-Order Multidomain Spectral Difference Method for the Navier-Stokes Equations on Unstructured Hexahedral Grids

Yuzhi Sun[1], Z. J. Wang[1,*] and Yen Liu[2]

[1] *Department of Aerospace Engineering, Iowa State University, 2271 Howe Hall, Ames, IA 50011, USA.*
[2] *NASA Ames Research Center, Mail Stop T27B-1, Moffett Field, CA 94035, USA.*

Received 15 May 2006; Accepted (in revised version) 9 August 2006

Available online 30 September 2006

**Abstract.** A high order multidomain spectral difference method has been developed for the three dimensional Navier-Stokes equations on unstructured hexahedral grids. The method is easy to implement since it involves one-dimensional operations only, and does not involve surface or volume integrals. Universal reconstructions are obtained by distributing solution and flux points in a geometrically similar manner in a unit cube. The concepts of the Riemann solver and high-order local representations are applied to achieve conservation and high order accuracy. In this paper, accuracy studies are performed to numerically verify the order of accuracy using flow problems with analytical solutions. High order of accuracy and spectral convergence are obtained for the propagation of an isotropic vortex and Couette flow. The capability of the method for both inviscid and viscous flow problems with curved boundaries is also demonstrated.

**AMS subject classfications**: 65M70, 76M20, 76M22

**Key words**: High order, unstructured grids, spectral difference, Navier-Stokes.

## 1 Introduction

It is well known that the computation of the aerodynamic flow field around a helicopter is a considerable challenge [10] because of the following difficulties: the strong interaction between the moving blades, vortices and wakes; the disparate length scales in the flow turbulence under flight conditions; the complex and moving geometries. Although low order (first and second order) finite volume methods have become the main choice of

*Corresponding author. *Email addresses:* sunyuzhi@iastate.edu (Y. Sun), zjw@iastate.edu (Z. J. Wang), Yen.Liu@nasa.gov (Y. Liu)

 

many commercial Computational Fluid Dynamics (CFD) codes, and proven successful in tackling a wide variety of flow problems [5, 11, 24] in engineering design, many in the CFD community believe vortex-dominated flows require high-order methods. This is mainly because lower order methods usually dissipate propagating vortices too quickly. For example, to study the classical blade-vortex interaction problem, it is estimated that the blade vortex should be well preserved for several revolutions. However, low order methods may completely “consume” the vortex after one or two revolutions, therefore making the computation results very inaccurate.

Many high-order methods (order $> 2$) have been developed in CFD for a wide range of applications, such as large eddy simulation, direct numerical simulation, computational aeroacoustics, etc. Most high-order methods were developed for structured grids, e.g., ENO/WENO methods [29], compact methods [18, 34, 35], optimized methods [32]. In particular, high-order compact methods have been successfully employed to tackle vortex dominated problems, including vortex breakdown over a delta wing [35]. For complex configurations, it is often very time-consuming to generate smooth structured grids required by these high-order methods. Our focus in this study is therefore on high-order methods for unstructured grids, and in particular unstructured hexahedral grids.

There have been intensive research efforts on high-order methods for flow simulation on unstructured grids in the last two decades. An incomplete list of notable examples includes the spectral element method [25], multi-domain spectral method [16,17], k-exact finite volume method [3], WENO methods [13], discontinuous Galerkin method [4, 7, 8], high-order residual distribution methods [1], spectral volume (SV) [21, 31, 36, 37] and spectral difference (SD) methods [14,19,20,23,38]. Among those methods, some are based on the weighted residual form of the governing equations, for instance the discontinuous Galerkin (DG) method. Some are based on the integral form of the governing equations, e.g., the k-exact finite volume method and the SV methods. Others, such as staggered grid multi-domain spectral method and SD method are based on the differential form.

When selecting a method to implement for three-dimensional problems, the cost and the complexity of the method is often an important factor. It is obvious that methods based on the differential form are the easiest to implement since they do not involve surface or volume integrals. This is particularly true when high-order curved boundaries need to be dealt with. Based on our experiences with the DG, SV and SD methods on 2D triangular meshes, the SD method seems the easiest to implement and most efficient for the 2D Euler equations. Therefore in the present study, the SD method is selected to solve the 3D Navier-Stokes equations on unstructured hexahedral grids. The use of hexahedral grids is again a compromise between flexibility and efficiency. Although tetrahedral grids are easier to generate for complex 3D configurations, hexahedral grids have been shown to possess higher efficiency and accuracy for viscous boundary layers [22].

The SD method and the staggered-grid multi-domain spectral method on hexahedral grids actually converge to the same method. The solution unknowns or degrees-of-freedom (DOFs) are the conserved variables at the Gauss points, while fluxes are evalu-